Assignments for Math 244 for Fall 2016 (part 4)

for Wednesday, November 9

1. The take-home portion of the third exam is due at the beginning of class.
2. There is no particular assignment. We will be starting Chapter 6 on this day. The prerequisite knowledge that is assumed here consists primarily of an understanding of improper integrals over infinite intervals, an ability to find partial fraction decompositions, and remembering how to perform integration by parts. If any of these topics are unfamiliar to you, I strongly urge you to review them before this class meeting. You might also benefit from reading the first three pages of Section 6.1.

for Friday, November 11

1. Read Section 6.1 of the text.
2. Do problems 1, 2, 5a, 9, 10, 14, 15, 17, 21, 26, and 27 in Section 6.1. You will need to become familiar with functions such as those in problems 1 and 2 so don't ignore them. For problem 5a, you will need to use integration by parts (and remember to use correct notation for improper integrals). For problems 9 and 10, you should use the linearity of the Laplace transform and previous results; there should be no integration. Problem 15 is another integration by parts problem, very similar to problem 5a. Finally, problem 26 introduces you to an important function, namely, the gamma function.
3. Turn in solutions to problems 15, 17, and 27a from Section 6.1. For problem 17, you should use the linearity of the Laplace transform and the result of problem 15. Problem 15 involves integration by parts and problem 27a involves a change of variables.

for Monday, November 14

1. Read Section 6.2 of the text.
2. Do problems 1 through 12 in Section 6.2. You may decide not to do all of these problems; just be aware that doing these types of problems quickly and accurately will be important for this chapter.
3. Turn in solutions to the following two problems related to the ideas in Section 6.2.
   i. Find $f(t)$ if the Laplace transform of $f(t)$ is $F(s) = \dfrac{3s-1}{s^2+4s+29}$.
   ii. Use the method of Laplace transforms to solve the initial value problem

$$y'' - 4y' - 5y = 0, \quad y(0) = 1, \quad y'(0) = 2.$$

for Wednesday, November 16

1. Read Section 6.2 again if necessary. Become very familiar with the table of Laplace transforms on page 317 and commit the basic Laplace transforms to memory; you need to be able to find transforms and inverse transforms quickly.
2. Do problems 13, 14, 16, 22, 28, 30, 33, and 37 in Section 6.2. Problems 28 and 37 may look scary, but they are not if you read carefully. Problems 30 and 33 then follow easily from problem 28.
3. Turn in solutions for problems 23 and 37 from Section 6.2. For problem 23, after solving for $Y(s)$, you can probably obtain the partial fraction decomposition using some creative but simple algebra; there is no need to put in constants A, B, and C and solve equations to find these values. However, you can always use this method as a default. For problem 37, begin by computing $g'(t)$ and $g(0)$.
4. As you do these problems, remember that one of the skills you can acquire through these problems is patience and attention to detail. This is a skill that is applicable in many areas of life and work. (Think about, for instance, a surgeon, a mechanic servicing an airplane, or someone testing the safety of a drug.)

for Friday, November 18

1. Read Section 6.3 of the text.
2. Do problems 1, 2, 3, 5, 7, 8, 10, 13, 14, 15, 17, 20, 21, 23, 24, 25ab, 30, 31, 35, and 37 in Section 6.3. This is a lot of problems but most of them should go quickly. If time is short, you can wait and do some of these over the break.
3. Turn in solutions to problem 37 from Section 6.3 (note that you need to use the result found in problem 34) as well as solutions to the two additional problems given below.
   i. Find the Laplace transform of the function f defined by $f(t) = u_1(t)\,t - u_4(t)\,(t-2)$. In addition, sketch a graph of the function f.
   ii. Find the inverse Laplace transform of the function F defined by $F(s) = \dfrac{(s-3)e^{-2s}}{s^2+2s+10}$.

for Monday, November 28

1. Read Section 6.4 of the text.
2. Do problems 1a, 3a, 5a, 9a, 13a, 14, and 15 in Section 6.4. For problem 9, show (algebraically) that the amplitude of the solution for $t > 6$ is $|\cos 3|$. It would be a good idea to look at some of the graphs of the solutions for the first few problems using Maple or some other graphing device.
3. Turn in solutions for problem 6a in Section 6.4 and for the following problem:

   Solve the IVP $y'' + y = f(t),\ y(0) = 0,\ y'(0) = 0$, where $f(t) = \begin{cases} 0, & \text{if } t < 0; \\ t, & \text{if } 0 \le t < 2; \\ 4-t, & \text{if } 2 \le t < 4; \\ 0, & \text{if } t \ge 4. \end{cases}$

   After determining the solution, find the amplitude of the resulting harmonic motion for $t \ge 4$. If possible, find the exact value for this amplitude, otherwise (with the deduction of one homework point) express the answer to the nearest ten-thousandth.

for Wednesday, November 30

1. Read Section 6.5 of the text. It is rather short and spends most of its time explaining the Dirac delta function. Once you get the basic idea and are willing to believe that such a "function" can be used, the examples are rather easy computationally.
2. Do problems 1, 6, 7, 8, and 14ab in Section 6.5. For problem 7, ignore that strangely placed $\cos t$.
3. Turn in a solution for the following problem:

   Consider the initial value problem $y'' + 2y' + 5y = 5\sin t + 4\,\delta(t - \frac{5}{2}\pi),\ y(0) = 1,\ y'(0) = 1$. Find the solution to this IVP, then determine the maximum value of the solution for $t > 0$. Give both the time at which the maximum value occurs and the value of y at this time, both correct to four decimal places. You will need the aid of an electronic device for the numerical results.

for Friday, December 2

1. Read (or perhaps skim) Section 6.6 of the text.
2. Do problems 2, 3, 4, 8, 12, 15, and 17 in Section 6.6.
3. Turn in solutions for problem 3 and the following problem:
   i. Find $t * t$ two ways; one using the definition and the other using Laplace transforms.
4. You may want to review Chapter 6 in preparation for the test next Wednesday. Make certain you are familiar with formulas for finding basic Laplace transforms (in both directions) as you will need to know them for the exam.

for Monday, December 5

1. You may want to look at an example or two in Sections 7.1 and 7.5, but you should not worry too much about reading these sections unless you are interested in how systems of differential equations appear in applications. The examples in class have hopefully given you sufficient background to tackle the problems listed below.
2. Do problems 5 and 7ab in Section 7.1 and problems 2, 5, and 16 in Section 7.5. If you have had some linear algebra, you should also do problem 17 in Section 7.5.
3. We will review for the exam in class on Monday.

for Wednesday, December 7

1. We have an exam on the topics we have covered in Chapter 6 along with our brief introduction to systems of differential equations. You do need to be familiar with the table of Laplace transforms and know how to use the definition to derive the entries in the table.
2. Exams on this material from previous semesters can be found on the website. At some point, you can use the following five problems as part of your review for the exam. You should do these problems without the textbook or a calculator and keep track of how long they take (that is, mimic a testing situation). The answers are given below. Recall that correcting your solution after checking the answer indicates that you would have been incorrect on the exam. These problems are on the order of moderate difficulty for an exam.

   i. Consider the initial value problem

   $$y'' + y = u_\pi(t) + 4\delta(t - 2\pi) - u_{4\pi}(t), \quad y(0) = 1, \quad y'(0) = 0.$$

   Solve this initial value problem, then find the amplitude of the solution for $t > 4\pi$.

   ii. Find the Laplace transform of the function f defined by $f(t) = u_4(t)\, t^2$.

   iii. Find the function f whose Laplace transform is given by $F(s) = \dfrac{(s^2 - 3s + 1)e^{-2s}}{s(s^2 - 1)}$.

   iv. Find the Laplace transform for the function f defined by

   $$f(t) = \begin{cases} (1 - \cos t)/t, & \text{if } t \neq 0; \\ 0, & \text{if } t = 0. \end{cases}$$

   Hint: Consider the function $tf(t)$ and use entry 19 in the table of Laplace transforms.

   v. Suppose that the Laplace transform of a function $h(t)$ is represented by $H(s)$. Given that $h(0) = 1$, $h'(0) = 2$, and $h''(0) = 3$, find (in terms of H) the Laplace transform for the function ϕ defined by

   $$\phi = h''' - 2h'' + 5h' - 4h.$$

i. For $t > 4\pi$, the solution is $y(t) = 3\cos t + 4\sin t$. The amplitude is thus 5.

ii. The Laplace transform F of f is defined by $F(s) = \dfrac{2}{s^3}\big(8s^2 + 4s + 1\big)e^{-4s}$.

iii. The function f is given by $f(t) = -\dfrac{1}{2}\, u_2(t)\big(2 + e^{t-2} - 5e^{2-t}\big)$.

iv. The Laplace transform F of f is defined by $F(s) = \ln \dfrac{\sqrt{s^2+1}}{s}$.

v. The Laplace transform Φ of ϕ is defined by $\Phi(s) = (s-1)(s^2 - s + 4)H(s) - s^2 - 4$.