Assignments for Math 126, Spring 2024

(due on the given date)

(1/17)
1. Read the lengthy email I sent to you about this course and look over some of the website links.
2. Read the syllabus (available on the website) very carefully; this may take 20 minutes or so.

(1/19)
1. Look over the syllabus again to pick up any tidbits you missed the first time.
2. Do the practice problems for differential calculus (refer to the website), reviewing as necessary.
3. Turn in answers to homework assignment 0 and solutions for homework assignment 1. You should model your solutions based on the sample homework solutions posted on the class website. As mentioned in the syllabus, it is recommended that you sketch a solution on a separate page before actually writing your final version on the homework page handout.
4. Carefully read the introduction to Chapter 2. Spend 5–10 minutes looking over Section 2.1.

(1/22)
1. Read Section 2.1 carefully.
2. Do problem 1, 2, 3, 4, 5, and 6 in Section 2.1.
3. Turn in solutions for homework assignment 2. Remember to record a more polished solution on the homework page handout.
4. Spend 5–10 minutes looking over Sections 2.2 and 2.3 of the textbook.

(1/24)
1. Read Sections 2.2 and 2.3 carefully.
2. Do problem 1, 2, and 5 in Section 2.2. For problem 2, draw a careful sketch of the region and look for areas you know how to find. For problem 5, it is best to draw a graph that represents the requested area and then use formulas for areas of triangles, trapezoids, or circles.
3. Do problems 1 and 2 in Section 2.3.
4. Turn in solutions for homework assignment 3. Remember to record a more polished solution on the homework page handout.
5. Spend 5–10 minutes looking over Sections 2.4 and 2.5 of the textbook.

(1/26)
1. Look over Sections 2.4 and 2.5 carefully, focusing on the ideas we discussed in class.
2. Do problems 3 and 5 (remember to use area for these problems) in Section 2.4.
3. Do problems 1bc, 2, 3, 4ab, and 7 in Section 2.5. Do be careful with your notation (remember dx) and be aware of the linearity properties of the integral that you are using.
4. Turn in solutions for homework assignment 4. Remember to record a more polished solution on the homework page handout.
5. Spend 5–10 minutes looking over Section 2.6 of the textbook.

(1/29) 1. Read Section 2.6 carefully.

2. Do problems 1acdefh, 2, 3, 4, 5, and 6 in Section 2.6. You will find that some of these problems force you to think outside the box; do not give up on them too quickly.
3. Turn in solutions for homework assignment 5.
4. Spend 5–10 minutes looking over Section 2.7 of the textbook.

(1/31) 1. Read Section 2.7 carefully.

2. Do problems 1, 2, and 3 in Section 2.7. Remember that we are not using any techniques of integration; we are simply thinking about differentiation in reverse. Since calculators will not be allowed on the exam, you should do all of these problems without the aid of an electronic device.
3. Turn in solutions for homework assignment 6.

(2/2) 1. Read Section 2.8 carefully.

2. Do the problems in Section 2.8; there are many integrals here but doing as many of them as possible is good practice. You should focus on learning to quickly recognize the general form of the antiderivative.
3. Turn in solutions for homework assignment 7.

(2/5) 1. Read Section 2.9 carefully.

2. You should look over the integrals in problem 1 and think about how you would solve each of them. Can you just write down the answer? Can you use guess and check? Do you need to make a substitution and, if so, what would u be? After making this assessment, do a few of each type beginning with 1b, 1e, 1i, 1j, 1k, and 1l. Repeat this process for problem 2, then begin with 2a, 2b, and 2e. Remember to change the limits for the definite integrals when using u-substitution; this avoids having to return to the x version of the antiderivative. Problems 3 and 4 indicate that there is more than one way to find an antiderivative while problem 5 shows how to determine the formula for the area of an ellipse. You can omit problem 6.
3. Turn in solutions for homework assignment 8.

(2/7) 1. Read Section 2.10 carefully.

2. Do problems 1b, 1c, 1e, 1i, 1j, 2b, 2c, 2d, 3, and 4 in Section 2.10.
3. Look over Section 2.11 as well as the extra notes for this section of Chapter 2.
4. Do problems 1a, 1d, 1e, and 1f in Section 2.11.
5. Turn in solutions for homework assignment 9.
6. We will review for the exam on this day.

(2/9) 1. We have our first exam this day, covering Sections 2.1 through 2.11.

2. The questions on the test will be similar to the homework problems you have been doing the past few weeks. You can find exams (and solutions) from a previous semester on the class website. However, it is important to remember that this is NOT a practice exam; our exam may look rather different than this one. In addition to the problems that were assigned in Sections 2.1 to 2.11 (for the record, doing those problems again without consulting your notes can be helpful), you can try problems 9, 10, and 12 in Section 2.24. For further practice evaluating integrals using ideas that we have discussed thus far, you can look over the integrals at the link 'Basic integration problems' on the website. You do not have time to do all of these integrals, but you should be able to determine the best approach for each one (that is, how to start the problem) within 30 seconds; you can stop there since you should have enough practice with the details by now.

3. No calculators or electronic devices will be allowed during the exam. You should plan your morning so that there is no need to leave the classroom during the exam. However, if it is necessary to do so, I ask that you leave your phone on your desk as you go. You will have 55 minutes for the exam. It is a good idea to show up a few minutes early if possible so that you are completely ready to begin the exam at the top of the hour.

4. You need to be able to state the definition of the derivative (Definition 1.7 in Section 1.7), the definition of the integral (Definition 2.1 in Section 2.3), and both parts of the Fundamental Theorem of Calculus (Theorem 2.4 in Section 2.6 and Theorem 2.5 in Section 2.7). All of these statements involve knowing all of the words, not just a few symbols. For the FTC, you should be aware of the focus of each part of the theorem and which mathematician is more closely linked to each version. You also need to know and be able to use the basic antiderivative formulas and the techniques of integration we have been practicing the last few sections.

(2/12) 1. There is no new material to read for this assignment.

2. In preparation for the applications of integration, you should look carefully at Exercise 2 in Section 2.2 (you may use the FTC in your solution) and Exercise 5 in Section 2.9. These problems indicate the sort of thinking that you will need to develop over the next few weeks.

(2/14) 1. Read Section 2.12 carefully.

2. Do the problems in Section 2.12. For some of these, sketch a careful graph and think about the problem before you set up the integrals.

3. Turn in solutions for homework assignment 10.

(2/16) 1. Read Section 2.13 carefully.

2. Do problems 1, 2, 4, 6, 7, and 9 in Section 2.13.

3. Turn in solutions for homework assignment 11.

(2/19) 1. There is no class today due to the President's Day holiday.

(2/21) 1. Read Section 2.14 carefully.

2. Do problems 1, 3, 4, 7, and 9 in Section 2.14. You will need to use a calculator for the second part of problem 9.
3. Turn in solutions for homework assignment 12.

(2/23) 1. Read Section 2.15 carefully.

2. Do problems 2 and 3 in Section 2.15.
3. Turn in solutions for homework assignment 13.

(2/26) 1. Read Section 2.18 carefully.

2. Do problems 2 and 3bc in Section 2.18.
3. Turn in solutions for homework assignment 15.

(2/28) 1. Read Section 2.17 carefully.

2. Do problems 3, 5, 8, and 13 in Section 2.17. Problem 13 is a good example of a multiple step problem. You may use the following fact for this problem:
   The distance from the point (x_0, y_0) to the line $Ax + By + C = 0$ is given by $\dfrac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$.
3. Turn in solutions for homework assignment 14.

(3/1) 1. Read Section 2.19 carefully.

2. Do the problems in Section 2.19, perhaps just doing enough of the problems to make sure that you know how to get started even if you do not carry out all of the details. Remember that these problems involve more algebra than calculus.
3. Turn in solutions for homework assignment 16.

(3/4) 1. Read Section 2.20 carefully.

2. Do problems 1abefgh, 2, 4, 5, and 6abf in Section 2.20.
3. Turn in solutions for homework assignment 17.

(3/6) 1. Read Section 2.21 carefully.

2. Do (or at least make the appropriate trig substitution and determine the new integral involving trig functions) problems 1abcef and 2bcde in Section 2.21.
3. Turn in solutions for homework assignment 18.
2. We will review for the exam on this day.

(3/8) 1. We have our second exam this day, covering Sections 2.12 through 2.21, omitting Section 2.16.

2. The questions on the test will be similar to the homework problems you have been doing the past few weeks. However, for the applications, the focus will be on setting up the appropriate integrals rather than evaluating them. You can find exams (and solutions) from previous semesters on the course website. However, it is important to remember that these are NOT practice exams; our exam may be different than these. In addition to the problems that were assigned in the relevant sections (for the record, doing those problems again without consulting your notes can be helpful), you can try problems 23, 24, 26, 34, 35, 36, and 37 in Section 2.24. For problems 36 and 37, focus on setting up the appropriate integral to determine the requested quantity.

3. No calculators or electronic devices will be allowed during the exam. You should plan your morning so that there is no need to leave the classroom during the exam. However, if it is necessary to do so, I ask that you leave your phone on your desk as you go. You will have 55 minutes for the exam. It is a good idea to show up a few minutes early if possible so that you are completely ready to begin the exam at the top of the hour.

4. You need to be able to state the definition of the derivative (Definition 1.7 in Section 1.7), the definition of the integral (Definition 2.1 in Section 2.3), and both parts of the Fundamental Theorem of Calculus (Theorem 2.4 in Section 2.6 and Theorem 2.5 in Section 2.7). All of these statements involve knowing all of the words, not just a few symbols. For the FTC, you should be aware of the focus of each part of the theorem and which mathematician is more closely linked to each version. You also need to know and be able to use the basic antiderivative formulas and techniques of integration that we learned at the beginning of the semester and, of course, you should be prepared to evaluate integrals using the new techniques in Sections 2.19, 2.20, and 2.21.

5. Two of the questions on the exam will ask for statements mentioned in the above item (4). In addition, one of the following five problems will be on the exam.

   i. Exercise 2 in Section 2.2 (an area problem)
   ii. Exercise 6 in Section 2.13
   iii. Exercise 7 in Section 2.14 (just the R part)
   iv. Exercise 2d in Section 2.15
   v. Exercise 8 in Section 2.17

   These textbook problems have been assigned over the past few weeks and numerical answers for these problems can be found in the appendix. I will NOT be doing these problems in class, but you may ask questions about them during office hours.

(3/25) 1. Carefully read the prelude to Chapter 3 (available on the website).

2. The material in Chapter 3 is more challenging than integration. As mentioned in the prelude, be prepared to think about mathematics differently over the next few weeks.

(3/27) 1. Read Section 3.1 carefully.

2. Do problems 1, 2, 3, 4, and 5 in Section 3.1.
3. Read the first two pages of the Model Induction Proofs (see the Prelude to Chapter 3 for the appropriate link). You should be able to find two errors in each of the incorrect proofs. You will also find a solution to problem 4 there, but you should try the problem on your own first.
4. Turn in solutions for homework assignment 19.

(3/29) 1. Reread Section 3.1 if necessary. Spend some time thinking about Fibonacci numbers.

2. Do problems 6 and 7 in Section 3.1. If you have not already done so, you should read the extra notes for Section 3.1, thinking carefully about each sentence of the examples given there.
3. Turn in solutions for homework assignment 20.

(4/1) 1. Read Section 3.2 carefully.

2. Do the problems in Section 3.2. Some of these problems will go quickly, but other problems may require some careful thought. Remember that there is a very strong emphasis on thinking about concepts as you study this material. You may find the extra notes for the sections in Chapter 3 enlightening. Read these slowly and try to understand every phrase and equation, asking questions on things that you do not understand.
3. Turn in solutions for homework assignment 21.

(4/3) 1. Read Section 3.3 carefully; work mindfully and deliberately on your technical reading skills.

2. Do problems 1–5 in Section 3.3. For problem 1, you need to write out careful steps as in one of the examples presented in the reading for this section. For the other problems, you will want to do some algebra and/or use the results in Theorem 3.8 and/or use the Squeeze Theorem. The extra notes may provide some helpful examples.
3. Turn in solutions for homework assignment 22.

(4/5) 1. Read Section 3.4 carefully.

2. Do problems 2–7 in Section 3.4.
3. Turn in solutions for homework assignment 23.

(4/8) 1. Read Section 3.5 carefully.

2. Do the problems in Section 3.5. Think carefully about each problem type.
3. Turn in solutions for homework assignment 24.

(4/10) 1. Read Section 3.6 carefully, making certain you understand the inequalities next to the graph.

2. Do the problems in Section 3.6. Make certain that you use correct notation for improper integrals and use these problems as an opportunity to review some integration.
3. Turn in solutions for homework assignment 25.

(4/12) 1. Read Section 3.7 carefully.

2. Do the problems in Section 3.7. When using the Comparison Test, it is important that you be very careful with the inequalities that you use; check them a second time! Problem 4 requires you to make some estimates about the sizes of the numbers (as we have done with other sums); as a start, determine how many two digit integers do not contain a 0 and how many three digit integers do not contain a 0. This is a good example of a nonroutine problem involving ideas that you have learned recently.

3. Turn in solutions for homework assignment 26.

(4/15) 1. Read Section 3.8 carefully.

2. Do the problems 1–8 in Section 3.8. For problems 1 and 2, it might be best to look at all of the series first, deciding what steps you would need to take to solve the problem. You can then fill in a few details to make certain you know how to write your solutions. Problems 4 through 7 help you think about the concepts; take them seriously. Problem 8 is important since you must first decide which of the convergence tests to use. Even if you do not carry out all of the details, think carefully about what the series does and which test to use to verify your conjecture.

3. Turn in solutions for homework assignment 27.

(4/17) 1. Read Section 3.9 carefully, paying particular attention to the last paragraph.

2. Do problems 2, 3, and 6 in Section 3.9. For problem 3, think carefully about the last paragraph in the section as you decide which test to use to check for convergence. Problem 6 is important for the material in the next few sections.

3. Turn in solutions for homework assignment 28.

(4/19) 1. Read Section 3.10 carefully.

2. Do the problems in Section 3.10; you may skip 2b and 7b. For problem 2, be certain to check the endpoints carefully. Be certain to understand the ideas behind problems 3 and 4. For problem 6, refer thoughtfully to the prototypes in problem 5.

3. Turn in solutions for homework assignment 29.

(4/22) 1. Read Section 3.11 carefully, pondering this new type of function.

2. Do problems 1, 2, and 4ab in Section 3.11.

3. Turn in solutions for homework assignment 30.

(4/24) 1. Read Section 3.12 carefully.

2. Do problems 1, 2, 3a, and 5 in Section 3.12 as well as problems 21a, 28, and 29 in Section 3.14.

3. Turn in solutions for homework assignment 31.

4. We will review for the exam on this day; see the next assignment for details.

(4/26) 1. We have an exam covering the first 12 sections of Chapter 3. See the next item for details.

2. Go over the sections we have covered and review the key concepts. You need to pay careful attention to the definitions and theorems, making certain that you understand them and remember when and how to use them. Practice some of the problems we have been doing, that is, do some of the problems again without looking at your previous solutions. Look at the exam on this material from a previous year (found on the website at the link 'Exams from Spring 2011') and be certain that the problems make sense. Note that some of the questions on the exam ask for examples of sequences or series with specific properties. As we have seen, using correct notation is extremely important for sequences and series so go over your turned-in homework solutions to identify any such errors you may have made. For additional problems, you can work on problems 1, 6, 11, 14, 21b, 24c, and 28 in Section 3.14. When you feel you are ready, take the diagnostic quiz on Chapter 3. This quiz can be found on the website, along with complete solutions.

3. Things to know for the exam include how to do a proof by induction, adjectives for sequences, limits of certain forms of sequences, the Completeness Axiom, the two sequences associated with a series, geometric series (and their sums) and p-series, all seven tests for convergence, the distinction between absolute convergence and conditional convergence, and how to work with power series. You also need to know the Maclaurin series for e^x, $\sin x$, and $\cos x$.

(4/29) 1. We will return to Chapter 2 and learn about another important integration technique.

2. You may want to review some of the basic integration formulas and techniques (such as substitution and integration by parts), and you should look carefully at the problems in Section 2.19. Doing so will also be good review for the final exam.

(5/1) 1. Read Section 2.22 carefully.

2. Do problems 1a,b,c,d,g,h,k in Section 2.22.

3. Turn in solutions for homework assignment 32.

(5/3) 1. Read Section 1.21 (not a typo) carefully.

2. Do problems 1, 2, and 3 in Section 1.21.

3. Turn in solutions for homework assignment 33.

(5/6) 1. Read Section 1.22 (not a typo) carefully.

2. Do problems 2, 4, 6, 8, and 9 in Section 1.22.

3. Turn in solutions for homework assignment 34.

4. Your work on Third Exam: Second Chance is due at the beginning of class.

5. We will review for the final exam on this day; see the next pages for further information.

As you may recall from reading the syllabus, the goals for this course are

- to develop quantitative reasoning skills;
- to learn how to read technical material; [reading the textbook and notes on your own]
- to learn to write technical information correctly and clearly; [via feedback on collected HW problems]
- to take pride in your work and to avoid errors; [see item (4) on the syllabus]
- to learn how to solve non-routine problems; [see the second paragraph of the syllabus]
- to appreciate/understand how mathematicians view mathematics;
- to comprehend some aspects of calculus.

It is with these goals in mind that the final exam will be written. The exam is comprehensive and covers all of the sections that we have discussed this semester, with a slight emphasis on the more recent material. The final exam will require the skills and concepts that you have been practicing and pondering this semester. It is your responsibility to go back over the sections and make certain that you know how to do the types of problems we have encountered. One or two of the problems on the final exam will be more involved than the sorts of problems that have appeared on the other exams that we have had. This should not be that much of a surprise because most of the test questions have been somewhat easier and shorter than homework problems due to the time constraints of a 50 minute exam. It is now time for you to put all your knowledge together and show me what you have learned this semester.

Here is the (most likely) introduction to the final exam that you will be taking.

Write neat, concise, and accurate solutions to each of the problems in the space provided—I will not give any credit for steps I cannot follow. Your solutions should be written in the style expected for collected homework problems. Pay particular attention to correct use of notation and use complete sentences when appropriate. Each of the * problems is worth * points (making a total of 80). No electronic devices or calculators are allowed for this exam.

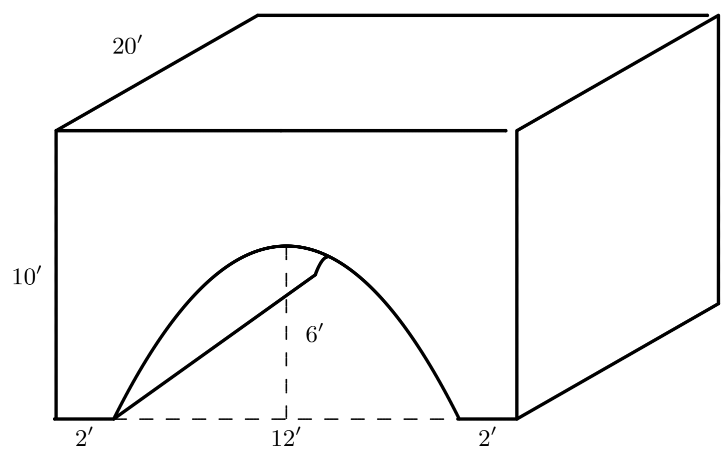
The problems will be graded as indicated in the heading so it is important that you work toward avoiding computational errors and that you pay attention to your writing and notation. The best advice is to review for the exam by looking over the sections in the text that we have covered, thinking carefully about the ideas we have discussed, and understanding the types of problems that have appeared on previous exams. You can redo the assignments problems (there were 34 of these), redo (or do?!) the assigned problems from the sections in the textbook, look over your previous exams, and work on some of the review problems that appear on the next few pages. It is important that you arrive at the exam with a refreshed mind and body, and be prepared to stay positive and work hard for two plus hours. As just indicated, although the exam is written for a two hour period, you may stay a while longer to avoid any time pressure. The exam time period is thus 9:00–11:30 ish on the appropriate day of our final exam. (Recall that the final exam schedule for all classes can be found on the Registrar's page.)

As should come as no surprise, it is expected that you can state (as well as understand) the definition of the derivative, the definition of the integral, and both versions of the Fundamental Theorem of Calculus. Do not lose points by ignoring this fact. You should also be familiar with basic integration formulas and techniques of integration, be able to solve problems involving applications of the integral, understand the main ideas behind sequences, series, and power series, and know the Maclaurin series for e^x, $\sin x$, and $\cos x$. This is just a sampling of the things that you need to know for the exam; if you have been keeping up during the semester, it should not be too difficult to remember the common formulas and ideas that we have been using somewhat regularly.

The following problems are not representative of the final exam. They are simply intended to give you some indication of the nature of a more difficult or novel problem that may appear on the final exam. Having said that, I do recommend that you give them some thought. However, keep in mind that most of the problems on the final exam will be (or at least should be) quite familiar to you. You can look at the final exam from Spring 2011 that is located on the course website but be aware that our final exam will not necessarily look like this.

1. Consider two different solids. The base of each solid is a triangle with vertices $(0,0)$, $(2,4)$, and $(6,0)$. For solid A, each cross-section perpendicular to the y-axis is an equilateral triangle. For solid B, each cross-section perpendicular to the x-axis is a square. Find the ratio of the volume of solid A to the volume of solid B.

2. Find the number of cubic yards of concrete necessary to construct the culvert shown below. Assume that the arch of the culvert (which is empty space) has a parabolic shape.



3. Let $a_1 = 2$ and $a_{n+1} = 3 - (1/a_n)$ for each positive integer $n \geq 1$. Use mathematical induction to prove that $a_n = \dfrac{f_{2n+1}}{f_{2n-1}}$ for each positive integer n, where f_n refers to the nth Fibonacci number.

4. For each positive integer n, let
$$y_n = \frac{1}{3n+2} + \frac{1}{3n+4} + \frac{1}{3n+6} + \cdots + \frac{1}{5n}.$$
Find the limit of the sequence $\{y_n\}$. (Try writing y_n in summation notation and think about integrals.)

5. Determine (using familiar calculus functions) the function represented by $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k (2k+1)}{4^k k!}\, x^{2k}$.

The problems that follow are more similar to the problems that you have been doing as homework. As you try these problems, put yourself in the mindset of an exam. That is, do not use your notes or look at the answer until you have finished the problem. Pay attention to problems you do not know how to start (these problems represent a lack of understanding) and problems you know how to start but get incorrect answers at the end (these problems indicate of lack of attention to detail).

Miscellaneous problems to try before the final exam

Since many of the answers are given right after the problem, you need to be careful to avoid using the answer as a hint for how to start the problem as this does not mimic a testing situation. Proceed without technology if at all possible. Omit problems marked with an asterisk.

1. Evaluate the limit $\lim_{n\to\infty} \dfrac{n^4+n^2+1}{3^3+6^3+9^3+\cdots+(3n)^3}$. (4/27)

2. Use an integral to evaluate the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \left(5+\dfrac{3i}{n}\right)^2 \dfrac{3}{n}$. (129)

3. Evaluate each of the following integrals.

   a) $\int_{-1}^{2} (2x-3)(x-1)\,dx$ b) $\int_{0}^{3} (4x+2|x-1|)\,dx$ c) $\int_{0}^{1} \left(2t-3+2\sqrt{1-t^2}\right)dt$

   d) $\int_{1}^{8} \dfrac{x+2}{\sqrt[3]{x}}\,dx$ e) $\int_{1}^{4} \dfrac{1}{3x-2}\,dx$ f) $\int_{0}^{2} \dfrac{1}{4+x^2}\,dx$

   (The values are 15/2, 23, $\dfrac{\pi}{2}-2$, 138/5, $\frac{1}{3}\ln 10$, and $\pi/8$, respectively.)

4. Use a simpler function to approximate $\int_{1}^{2} \dfrac{1}{\sqrt{4x^6-1}}\,dx$. Is your estimate high or low? (3/16, low)

5. Find the derivative of the function F defined by $F(x) = \int_{0}^{x^2} t\sqrt{t^3+4}\,dt$. $\left(F'(x) = 2x^3\sqrt{x^6+4}\,\right)$

6. Suppose that $v(t) = 3t - t^3$ gives the velocity in meters per second of a particle at time t seconds. Find the distance traveled by the particle for the time interval $0 \le t \le 4$. (44.5 meters)

7. Find the area of the region bounded by the curves $x^2y = 90$ and $40x + y = 130$. (40)

8. Find the area of the region bounded by the curves $y = 2\sqrt{x}$ and $y = x^3/16$. (20/3)

9. Find the volume of the solid that is generated when the region bounded by the curves $y = 4x$ and $y = x^2$ is revolved around (a) the x-axis (b) the y-axis. $\left(\left(\dfrac{2048}{15}\,\pi\right) \text{ and } \left(\dfrac{128}{3}\,\pi\right)\right)$

10. Suppose the base of a solid is the part of the parabola $y = 8 - 0.5x^2$ that lies above the x-axis and that each cross section perpendicular to the y-axis is a semicircle. Find the volume of this solid. (32π)

11. Find the volume of the solid that is generated when the region that lies below the curve $y = \ln x$ and above the x-axis on the interval $[1, e]$ is revolved around the y-axis. $\left(\pi(e^2+1)/2\right)$

12. Find the volume of the solid that is generated when the region that lies above the x-axis and below the curve $y = \sqrt{\dfrac{14(3-x)}{(x+1)(7-x)}}$ on the interval $[0, 3]$ is revolved around the x-axis. $\left(7\pi(\ln 16 - \ln 7)\right)$

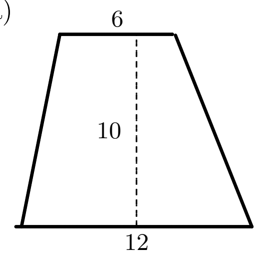
13. Find the center of mass of the region bounded by the curves $y = 2\sqrt{x}$ and $y = x^3/16$. $\left(\dfrac{48}{25}, \dfrac{12}{7}\right)$

14. Find the center of mass of the solid that is generated when the region below the curve $y = 4e^{-x/4}$ and above the x-axis on the interval $[0, \infty)$ is revolved around the x-axis. $(2, 0, 0)$

15. Find the length of the curve $y = 4x^{3/2}$ on the interval $[0, 10]$. (127)

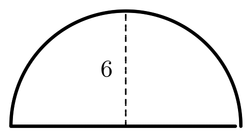
16. Find the force exerted by a liquid with weight density w on one side of each vertically submerged plate. The units on the figures are feet and the top of each plate is six feet beneath the surface of the liquid.

a)



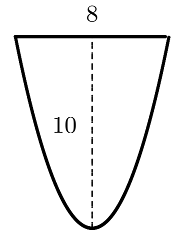
trapezoid

b)



semicircle

c)



parabola

(The forces are $1040w$, $(216\pi - 144)w$, and $1600w/3$ pounds, respectively.)

17. Evaluate each of the following definite integrals.

a) $\displaystyle\int_0^2 \frac{x}{4+x^2}\,dx$ b) $\displaystyle\int_0^2 \frac{x}{\sqrt{4+x^2}}\,dx$ c) $\displaystyle\int_0^2 \frac{x}{\sqrt{4+x}}\,dx$

d) $\displaystyle\int_1^\infty \frac{8}{(2x+5)^3}\,dx$ e) $\displaystyle\int_0^\infty \frac{6+e^{2x}}{e^{3x}}\,dx$ f) $\displaystyle\int_1^{\sqrt{2}} \frac{\sqrt{x^2-1}}{x^4}\,dx$

(The values are $\frac{1}{2}\ln 2$, $2\left(\sqrt{2}-1\right)$, $\dfrac{32}{3} - 4\sqrt{6}$, $2/49$, 3, and $\sqrt{2}/12$, respectively.)

18. * Use the trapezoid rule and Simpson's rule with $n = 4$ to approximate $\displaystyle\int_0^1 e^{-x^2/2}\,dx$ to four decimal places. (The approximations are 0.8526 and 0.8557, respectively.)

19. * Suppose that the following table represents the velocity of a particle moving in a straight line.

t	(sec)	0	1	2	3	4	5	6
v	(m/sec)	0	5	10	12	8	4	0

Use Simpson's rule to approximate the distance traveled by the particle. (40 meters)

20. Evaluate each of the following indefinite integrals.

a) $\displaystyle\int \left(2\sqrt{x}+1\right)^2 dx$ b) $\displaystyle\int \frac{3x}{(2x^2+5)^3}\,dx$

c) $\displaystyle\int \frac{12x}{(3x-1)^2}\,dx$ d) $\displaystyle\int \frac{3x+1}{\sqrt{12x-x^2}}\,dx$

e) $\displaystyle\int \frac{3x+8}{x^2+4x+6}\,dx$ f) $\displaystyle\int \frac{4x-7}{2x+1}\,dx$

g) $\displaystyle\int \arctan x\,dx$ h) $\displaystyle\int \frac{\sqrt{x^2+4}}{x^4}\,dx$

i) $\displaystyle\int \frac{3x+1}{\sqrt{13-12x-x^2}}\,dx$ j) $\displaystyle\int \frac{4x-1}{x^2+2x-15}\,dx$

k) $\displaystyle\int \frac{x^2+2x+4}{x^3+x^2+x+1}\,dx$ ℓ) $\displaystyle\int x\,e^{-x/2}\,dx$

m) $\displaystyle\int \frac{3x-7}{2x^2+7x-9}\,dx$ n) $\displaystyle\int \frac{2x^2+7x-9}{3x-7}\,dx$

The answers for these integrals are given below.

a) $2x^2 + \dfrac{8}{3}x^{3/2} + x + C$

b) $\dfrac{-3}{8(2x^2+5)^2} + C$

c) $\dfrac{4}{3}\left(\ln|3x-1| - \dfrac{1}{3x-1}\right) + C$

d) $-3\sqrt{12x - x^2} + 19\arcsin\left(\dfrac{x-6}{6}\right) + C$

e) $\dfrac{3}{2}\ln\left(x^2+4x+6\right) + \sqrt{2}\arctan\left(\dfrac{x+2}{\sqrt{2}}\right) + C$

f) $2x - \dfrac{9}{2}\ln|2x+1| + C$

g) $x\arctan x - \dfrac{1}{2}\ln(1+x^2) + C$

h) $\dfrac{-(x^2+4)^{3/2}}{12x^3} + C$

i) $-3\sqrt{13-12x-x^2} - 17\arcsin\left(\dfrac{x+6}{7}\right) + C$

j) $\dfrac{11}{8}\ln|x-3| + \dfrac{21}{8}\ln|x+5| + C$

k) $\dfrac{3}{2}\ln|x+1| - \dfrac{1}{4}\ln(x^2+1) + \dfrac{5}{2}\arctan x + C$

ℓ) $-2(x+2)e^{-x/2} + C$

m) $\dfrac{41}{22}\ln|2x+9| - \dfrac{4}{11}\ln|x-1| + C$

n) $\dfrac{1}{3}x^2 + \dfrac{35}{9}x + \dfrac{164}{27}\ln|3x-7| + C$

21. Prove the following statement: for each positive integer n, the integer $2^{5n-4} + 5^{2n-1}$ is divisible by 7.

22. Find the limit of the given sequence.

a) $\left\{\dfrac{k}{\sqrt{3k^2+4k+1}}\right\}$ b) $\left\{\sqrt{k^2-7k+15} - k\right\}$ c) $\left\{k\left(\sqrt[k]{10}-1\right)\right\}$

d) $\left\{\left(1-\dfrac{2}{3n}\right)^n\right\}$ e) $\left\{\dfrac{4^n+n^2}{2^{2n-3}+n^7}\right\}$ f) $\left\{\sqrt[n]{4n^2+n+3}\right\}$

(The limits are $1/\sqrt{3}$, $-7/2$, $\ln 10$, $e^{-2/3}$, 8, and 1.)

23. Define a sequence $\{x_n\}$ by $x_1 = 5$ and $x_{n+1} = 4 - (1/x_n)$ for $n \geq 1$. Prove that $1 \leq x_n \leq 5$ for all n, then prove that $\{x_n\}$ is a decreasing sequence. Conclude that $\{x_n\}$ converges and find its limit. $(2+\sqrt{3})$

24. Find the sum of the given series.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}3^k}{4^{k-1}}$ b) $\displaystyle\sum_{k=1}^{\infty} \frac{3^k+5^k}{7^k}$ c) $\displaystyle\sum_{k=1}^{\infty} \frac{(-4)^k}{(2k+1)!}$

(The sums of the series are 12/7, 13/4, and $\frac{1}{2}\sin 2 - 1$, respectively.)

25. Let $\displaystyle\sum_{k=1}^{\infty} a_k$ be an infinite series and suppose that its sequence $\{s_n\}$ of partial sums is given by $s_n = \dfrac{n+1}{1-3n}$ for all $n \geq 1$. Find a_1, a_2, a_{10}, and the sum of the series.

(The values are -1, $2/5$, $2/377$, and $-1/3$, respectively.)

26. Determine whether or not the given series converges.

a) $\displaystyle\sum_{k=1}^{\infty} \frac{12}{3k+2}$ b) $\displaystyle\sum_{k=1}^{\infty} \frac{4k-1}{k^2+5k+2}$ c) $\displaystyle\sum_{k=1}^{\infty} \frac{2k^2+3}{k^4+7k-1}$

d) $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{2^k+6^k}$ e) $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{\sqrt[k]{5}}$ f) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k}{3k+1}\right)^k$

(The series are D, D, C, C, D, and C, respectively.)

27. Classify the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-3)^k k!}{3 \cdot 7 \cdot 11 \cdot \cdots \cdot (4k-1)}$ as AC, CC, or D. (It is AC.)

28. Show that each of the series $\displaystyle\sum_{k=1}^{\infty} \frac{3^k \sin k}{4^k}$ and $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k^2}{k^4 + 3k^2 + 10}$ is absolutely convergent.

29. Carefully prove that each of the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{5k+4}$ and $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1} k}{k^2+1}$ is conditionally convergent.

30. Find the radius of convergence for the power series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2 3^k}(x-4)^k$. (3)

31. Find the interval of convergence for the power series $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)2^k}(x-1)^k$. $((-1, 3])$

32. Give an example of a power series with $[4, 10)$ as its interval of convergence.

    One example is $\displaystyle\sum_{k=0}^{\infty} \frac{1}{(2k+1)3^k}(x-7)^k$.

33. Find (in more familiar terms) the function represented by the power series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k}(x+1)^k$.

    This is the power series for the function $f(x) = \dfrac{x+1}{x+3}$, valid on the interval $(-3, 1)$.

34. Find the Maclaurin series for the function $f(x) = \dfrac{1}{5-2x}$ and determine its interval of convergence.

    The Maclaurin series is $\displaystyle\sum_{k=0}^{\infty} \frac{2^k}{5^{k+1}} x^k$, with interval of convergence $(-2.5, 2.5)$.

35. By differentiating an appropriate power series (see problem 3.11.2), find the sum of the series $\sum_{k=1}^{\infty} k^3 x^k$.

    $$\sum_{k=1}^{\infty} k^3 x^k = \frac{x + 4x^2 + x^3}{(1-x)^4}.$$

36. Use known series to find the Maclaurin series for the given function.

    a) $f(x) = e^{-x/3}$ **b)** $g(x) = \sin(x^2)$ **c)** $h(x) = \dfrac{1-\cos x}{x}$

    $$f(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{3^k k!} x^k, \qquad g(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{4k+2}, \qquad h(x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{(2k)!} x^{2k-1}$$

37. Use known Maclaurin series to determine in more familiar terms the given function. (See exercises 28 and 29 in Section 3.14 if you want more practice for these types of problems.)

    a) $\displaystyle\sum_{k=0}^{\infty} \frac{1}{2^k k!} x^k$ **b)** $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} x^{2k}$ **c)** $\displaystyle\sum_{k=0}^{\infty} \frac{(-9)^k}{(2k)!} x^{2k+1}$

    The functions are $e^{x/2}$, $\sin x/x$, and $x\cos(3x)$, respectively.

38. Find the Taylor series for the function $f(x) = 1/(2x-1)$ centered at $a = 6$.

    $$\frac{1}{2x-1} = \sum_{k=0}^{\infty} \frac{(-2)^k}{11^{k+1}}(x-6)^k \text{ with } \rho = 5.5.$$