Partial Review of Calculus I

It would be in your best interest to spend an hour or more flipping through the first chapter of the online textbook to refresh your memory on the topics and ideas discussed in Calculus I. We will not be spending any more time reviewing Calculus I so you need to hit the ground running. To be prepared for this class, the following problems should not be all that difficult after your review. (By the way, you should be able to do all of these problems without a calculator.)

1. Find an equation for the line tangent to the graph of $y = x^2 + 3x$ when $x = 1$.
2. Evaluate the limits $\displaystyle\lim_{x\to\infty} \frac{5x^4 + 3x^3 + 2}{2x^4 - 4x + 9}$, $\displaystyle\lim_{x\to\infty} \frac{(1-4x)(x^2+5)}{(x-1)(2x-3)(3x+4)}$, and $\displaystyle\lim_{x\to\infty} \frac{\sqrt{5x^4 + 3x^3 + 2}}{2x^2 + 4x + 9}$.
3. Evaluate the limits $\displaystyle\lim_{x\to 0} \frac{\tan(3x)}{e^{-7x} - 1}$, $\displaystyle\lim_{x\to 0} \frac{e^{2x} - 1}{x}$, and $\displaystyle\lim_{x\to 0} \frac{2^x - 1}{x}$.
4. Let $f(x) = x^3 + 6x$ and let $R(x) = \dfrac{f''(x)}{f'(x)}$. Find the exact (this means no decimal approximations) maximum value of the function R.
5. Let a be a positive constant and let f be the function defined by $f(x) = x^3 - 3a^2x$. Find the maximum and minimum values of f on the interval $[0, 3a]$.
6. Let a be a positive constant and consider the function f defined by $f(x) = e^{-x^2/a^2}$. Find the (x, y) coordinates of the inflection points of the graph of this function. Note that the y-coordinates of the inflection points are independent of the value of a.
7. At a certain instant, the radius of a sphere is 12 cm and its volume is increasing at a rate of 200π cm^3/sec. How fast is the diameter of the sphere increasing at this instant?
8. Let f be the function defined by $f(x) = x^2 + 2x$. Show that the point whose existence is guaranteed by the Mean Value Theorem for f on the generic interval $[a, b]$ is the midpoint of the interval.
9. Use the definition of the derivative to find the derivative of the function $f(x) = 1/x$.

   For the remaining problems, find and simplify (this part is very important) the derivative of the given function.
10. $f(x) = x \ln x - x$
11. $w = \sin z - \dfrac{1}{3} \sin^3 z$
12. $g(x) = \dfrac{x}{\sqrt{4 - x^2}}$
13. $h(t) = t - \dfrac{1}{\sqrt{2}} \arctan\left(\dfrac{t}{\sqrt{2}}\right)$
14. $y = \ln\left(x + \sqrt{x^2 + 1}\right)$

Answers for the review problems

1. An equation for the tangent line is $y = 5x - 1$. (Solutions to homework problems that are turned in should give a complete sentence for the conclusion.)

2. The limits are $5/2$, $-2/3$, and $\sqrt{5}/2$. You should be able to do the first two by inspection using a shortcut BUT be absolutely certain you understand why the shortcut works. L'Hôpital's Rule is not a good option for the third limit; you should have seen other ways to evaluate limits of this type. In this case, multiply numerator and denominator by $1/x^2$.)

3. The limits are $-3/7$, 2, and $\ln 2$.

4. The maximum value of R is $1/\sqrt{2}$. You should have done something to verify that this is the maximum value. (Note that the requested answer is the value of the function, not the x value that generates it.)

5. The maximum value of f is $18a^3$ (and it occurs when $x = 3a$) and the minimum value of f is $-2a^3$ (which occurs when $x = a$). See Section 1.12 for the best methodology for finding max/min values on a closed and bounded interval. (Once again, we are interested in the outputs of f, not the inputs.)

6. The inflection points for this curve are $\left(-\frac{a}{\sqrt{2}}, \frac{1}{\sqrt{e}}\right)$ and $\left(\frac{a}{\sqrt{2}}, \frac{1}{\sqrt{e}}\right)$. Make certain that you checked that you really do have inflection points; you cannot just solve $f''(x) = 0$ and arrive at a conclusion. (One of the purposes of this problem, as well as the previous problem, is to give you some practice working with parameters (the constant a) rather than just numbers.)

7. The diameter of the sphere is increasing at a rate of 25/36 cm/sec at the instant the radius is 12 cm. (This is a related rates problem and correct units for the solution are needed.)

8. This problem contains its own solution. Make certain you are familiar with the Mean Value Theorem.

9. You should know that $f'(x) = -1/x^2$, but the important part is being able to use the limit definition of the derivative to compute this function.

10. $f'(x) = \ln x$

11. $\dfrac{dw}{dz} = \cos^3 z$ (know simple trigonometric identities)

12. $g'(x) = \dfrac{4}{(4 - x^2)^{3/2}}$ (simplify carefully)

13. $h'(t) = \dfrac{t^2 + 1}{t^2 + 2}$ (did you remember the chain rule?)

14. $\dfrac{dy}{dx} = \dfrac{1}{\sqrt{x^2 + 1}}$ (more algebra practice)