for Wednesday, January 17

1. Attend class and please make sure that you have access to some version of the third edition of the textbook *Fundamentals of Complex Analysis with Applications to Engineering and Science* by Saff and Snider.

for Friday, January 19

1. Look over the syllabus and read Section 1.1.
2. Work on exercises 5 through 24 in Section 1.1. I will not be collecting any of these solutions.
3. Student presentations at the board for exercises in Section 1.1 are Warren (8), Jenner (12), Kai (17), Peter (20b), Feras (21), and Caroline (23/24).

for Monday, January 22

1. Read Section 1.2.
2. Work on exercises 4, 6, 7, 8, 13, 16, and 17 in Section 1.2.
3. Turn in carefully written solutions for exercises 6 and 16 in Section 1.2.
4. Student presentations at the board for exercises in Section 1.2 are Warren (17), Jenner (13), Kai (8), Peter (7g), Feras (7e), and Caroline (7c).

for Wednesday, January 24

1. Read Section 1.3 carefully.
2. Work on exercises 5, 6, 7, 11, 12, and 13 in Section 1.3.
3. Turn in a carefully written solution for exercise 11 in Section 1.3.
4. Student presentations at the board for exercises in Section 1.3 are Warren (7d), Jenner (6d), Kai (13a, give explicit counterexample), Peter (7h), Feras (13c, give explicit counterexample), and Caroline (6b).

for Friday, January 26

1. Read Section 1.4 carefully.
2. Work on exercises 1, 2, 3, 4, 8, 10, 11, 17, 18, and 21 in Section 1.4.
3. Turn in a carefully written solution for exercise 21 in Section 1.4.
4. Student presentations at the board for exercises in Section 1.4 are Warren (4b), Jenner (2a), Kai (1b), Peter (6a), Feras (4a), and Caroline (2c).

for Monday, January 29

1. Read Section 1.5 carefully.
2. Work on exercises 4, 5, 7bc, 10, and 16 in Section 1.5.
3. Student presentations at the board for exercises in Section 1.5 are Warren (5d), Jenner (16), Kai (10), Peter (5f), Feras (4a), and Caroline (7b).

for Wednesday, January 31

1. Read Section 1.6 carefully, paying particular attention to the definitions and graphs. Also, look over Section 1.7, but you do not need to bother with the details unless you find the topic particularly interesting.
2. Work on exercises 1–19 in Section 1.6, recording any questions you have about these exercises. If time permits, ponder exercises 2 and 6 in Section 1.7.
3. The first special assignment is due at the beginning of class on this day.

for Friday, February 2

1. Read Section 2.1.
2. Work on exercises 1–10 and 13 in Section 2.1.
3. Turn in solutions for exercises 4c and 6c (you do not need to verify the foci part).
4. Student presentations at the board for exercises in Section 2.1 are Warren (8), Jenner (11b), Kai (10d), Peter (13b), Feras (10a), and Caroline (11c).

for Monday, February 5

1. Read Section 2.2.
2. Work on exercises 1, 2, 4, 5, 7, 8, 11, 14, 20, and 21 in Section 2.2. Formal proofs are not required for exercises 7 and 11; just use "calculus" ideas. Note that there is an error in one of the limits in exercise 11; see if you can spot it. Exercises 20 and 21 are expected to go quickly given previous results.
3. Turn in solutions for exercises 2 (use a graph as part of your description), 8 (this is a formal δ-ϵ proof), and 11c (show the steps clearly and use correct notation).

for Wednesday, February 7

1. Read Section 2.3.
2. Work on exercises 4, 7abc, 9, 11, 13, 14, and 15 in Section 2.3.
3. Turn in solutions for exercises 11e and 11f, solving them by finding expressions that involve the variable $z = x + iy$ for these functions. As a hint for 11e, try to make the term $|z + i|$ appear. Also, give a careful proof of the quotient rule. The best way to proceed is to use the definition to find the derivative of the function $1/g(z)$, then use the product rule on $f(z) \cdot (1/g(z))$. In your proof for the derivative of the function $1/g(z)$, note where you use the continuity of the function g.

for Friday, February 9

1. Read Section 2.4.
2. Work on exercises 1, 2, 3, 5, 8, 9, 10, and 11 in Section 2.4.
3. Turn in solutions for exercises 2, 8, and 11.

for Monday, February 12

1. Read Section 2.5.
2. Work on exercises 1, 3abcd, 5, 6, and 18 in Section 2.5.
3. Spend some time reviewing the material that we have covered thus far in the course. We will go over the concepts in class and discuss any questions that you may have.

for Wednesday, February 14

1. We have a test covering Chapter 1 (omitting Section 1.7) and Sections 2.1 through 2.5. You should focus on the concepts we have considered and be able to perform simple calculations with complex numbers. Flipping through the sections we have covered and going over the problems that have been assigned should prepare you well for the exam.
2. To avoid the possible stress of time constraints, you may work on the exam until 12:15. If this creates a time conflict for you, please let me know.

for Friday, February 16

1. The second special assignment is due at the beginning of class on this day.
2. You might find it interesting to spend 20 to 30 minutes browsing Sections 2.6 and 2.7. The first section may help you understand how harmonic functions appear in applications, while the second section may give you some insight into fractals.

for Monday, February 19

1. There is no class today due to the President's Day holiday. However, it might be helpful for you to start your reading of the lengthy Section 3.1.

for Wednesday, February 21

1. Read Section 3.1 through the solution to Example 4.
2. Work on exercises 1, 3, 4, 5, 6, 7, 10, and 11 in Section 3.1; be prepared to discuss these in class.
3. Turn in a solution for exercise 10; you may find some inequalities on the FTA handout helpful.

Exercise 3.1.9: Show that if z_0 is a zero of order d for the polynomial P, then there are positive constants c_1 and c_2 such that $c_1|z-z_0|^d \le |P(z)| \le c_2|z-z_0|^d$ for all z sufficiently close to z_0.

Solution: Let $P(z) = (z-z_0)^d\, Q(z)$, where Q is a polynomial for which $Q(z_0) \neq 0$. Since Q is continuous at z_0 and $\frac{1}{2}|Q(z_0)| > 0$, there exists $\delta > 0$ such that $|Q(z) - Q(z_0)| < \frac{1}{2}|Q(z_0)|$ for all z that satisfy $|z - z_0| < \delta$. For these values of z, we see that

$$|Q(z)| \le |Q(z) - Q(z_0)| + |Q(z_0)| < \frac{1}{2}|Q(z_0)| + |Q(z_0)| = \frac{3}{2}|Q(z_0)|;$$
$$|Q(z_0)| \le |Q(z) - Q(z_0)| + |Q(z)| < \frac{1}{2}|Q(z_0)| + |Q(z)|.$$

It follows that

$$\frac{1}{2}|Q(z_0)| < |Q(z)| < \frac{3}{2}|Q(z_0)| \quad \text{and thus} \quad \frac{1}{2}|Q(z_0)|\,|z-z_0|^d \le |P(z)| \le \frac{3}{2}|Q(z_0)|\,|z-z_0|^d$$

for all z that satisfy $|z - z_0| < \delta$. ■

Problem: Prove that $\lim\limits_{z\to\infty} \left| \dfrac{a_{n-1}}{z} + \dfrac{a_{n-2}}{z^2} + \cdots + \dfrac{a_0}{z^n} \right| = 0.$

Solution: Let $S = \sum\limits_{k=0}^{n-1} |a_k|$. Given $\epsilon > 0$, choose $\rho = \max\{1, S/\epsilon\}$. For each complex number z that satisfies $|z| > \rho$, we find that

$$\begin{aligned}
\left| \frac{a_{n-1}}{z} + \frac{a_{n-2}}{z^2} + \cdots + \frac{a_0}{z^n} \right| &\le \frac{|a_{n-1}|}{|z|} + \frac{|a_{n-2}|}{|z|^2} + \cdots + \frac{|a_0|}{|z|^n} \\
&< \frac{|a_{n-1}|}{|z|} + \frac{|a_{n-2}|}{|z|} + \cdots + \frac{|a_0|}{|z|} \\
&= \frac{S}{|z|} < \frac{S}{\rho} \le \frac{S}{S/\epsilon} = \epsilon.
\end{aligned}$$

It follows that $\lim\limits_{z\to\infty} \left| \dfrac{a_{n-1}}{z} + \dfrac{a_{n-2}}{z^2} + \cdots + \dfrac{a_0}{z^n} \right| = 0.$ ■

Problem: Express the polynomial $P(z) = z^3 + 4z^2 - 3z - 7$ using powers of $(z + 2)$.

Solution: For one approach that only requires algebra, we note that

$$\begin{aligned}
Q(z) = P(z-2) &= (z-2)^3 + 4(z-2)^2 - 3(z-2) - 7 \\
&= (z^3 - 6z^2 + 12z - 8) + (4z^2 - 16z + 16) - (3z - 6) - 7 \\
&= z^3 - 2z^2 - 7z + 7.
\end{aligned}$$

It then follows that

$$P(z) = Q(z+2) = (z+2)^3 - 2(z+2)^2 - 7(z+2) + 7.$$

On the other hand, using the derivative formulas for Taylor polynomials, we note that

$$\begin{aligned}
P(z) &= z^3 + 4z^2 - 3z - 7; & & & P(-2) &= -8 + 16 + 6 - 7 = 7; \\
P'(z) &= 3z^2 + 8z - 3; & & \text{and} & P'(-2) &= 12 - 16 - 3 = -7; \\
P''(z) &= 6z + 8; & & & P''(-2) &= -12 + 8 = -4; \\
P'''(z) &= 6; & & & P'''(-2) &= 6.
\end{aligned}$$

We thus find that

$$P(z) = \frac{P(-2)}{0!} + \frac{P'(-2)}{1!}(z+2) + \frac{P''(-2)}{2!}(z+2)^2 + \frac{P'''(-2)}{3!}(z+2)^3 = 7 - 7(z+2) - 2(z+2)^2 + (z+2)^3,$$

the same polynomial obtained using the algebraic approach. ■

for Friday, February 23

1. Finish reading Section 3.1.
2. Work on exercises 12, 13, 14, 15, and 16 in Section 3.1.
3. Turn in solutions for exercise 16 (use the results in exercise 10) and the following problem:

   Use the formula in equation (21) to find the partial fraction decomposition for $R(z) = \dfrac{z^2 + 2iz + 3 + i}{z^4 + iz^3}$. Include the details (as simply as possible) of your computations.

for Monday, February 26

1. Read Section 3.2.
2. Work on exercises 5, 6, 7, 8, 10, 11, 12ac, 14ab, 17, and 20 in Section 3.2. For exercise 20, consider the outer domain path as starting at 1 then going to $1 + \pi i$, $-1 + \pi i$, -1, and back to 1. Trace the image of this path in the range.
3. Turn in solutions for exercises 8, 11 (also write out this function in $u(x, y)$ form), 12c, and 14b.

for Wednesday, February 28

1. Read Section 3.3.
2. Work on exercises 1, 3, 5, 6, and 9 in Section 3.3.
3. The third special assignment is due at the beginning of class on this day.

for Friday, March 1

1. Read Section 3.5; we will be skipping Sections 3.4 and 3.6.
2. Work on exercises 1, 3, 4, 10, 12, and 19 in Section 3.5.
3. Turn in solutions for exercises 10 and 19.

for Monday, March 4

1. Read Section 4.1, paying careful attention to the many definitions.
2. Work on exercises 1, 3, 4, 5, 6, 7, 8, 9, 10, and 11 in Section 4.1.
3. Turn in solutions for exercises 4 (find an appropriate parametrization), 8 (do not forget to do $-\Gamma$), and 10 (you need to write out $z(t)$ and use the arc length formula).

for Wednesday, March 6

1. Read Section 4.2, skimming as necessary.
2. Work on exercises 3abd, 5, 6, 8, 9, 10, 11, 12, and 14ac in Section 4.2. For 5 and 6, make extensive use of Example 2.
3. Turn in solutions for exercises 6c, 8, and 10.

for Friday, March 8

1. Read Section 4.3.
2. Work on exercises 1ace, 2, 6, 7, 9, and 12 in Section 4.3.
3. Turn in solutions for exercises 1ae (express the final answers in $a+bi$ form), 7, and 12. For exercise 7, include a picture that illustrates an example where $\mathrm{Log}\,(z-z_0)$ would not be an appropriate antiderivative. For exercise 12, locate the appropriate theorem from Section 4.2 needed in the proof. Note that this result is a variation on the Mean Value Theorem for real functions.

for Monday, March 25

1. Read Section 4.4b. You might also find it interesting to skim Section 4.4a for a different perspective.
2. Work on exercises 9, 10, 11, 13, 15, 16, 17, 18, and 19 in Section 4.4. Try to use partial fractions to give a much simpler solution for Exercise 18.
3. Turn in solutions for exercises 15 and 17.

for Wednesday, March 27

1. Read Section 4.5 and the extra notes for this section that were handed out in class. Focus on the results and keep track of the key theorems.
2. Work on exercises 1–11 in Section 4.5.
3. Turn in solutions for exercises 3d and 6.

for Friday, March 29

1. Read Section 4.6 and the extra notes for this section that were handed out in class.
2. Work on exercises 3, 5, 10, 11, and 17 in Section 4.6.
3. Turn in a solution for exercise 17.
4. The fourth special assignment is due at the beginning of class on this day.

for Monday, April 1

1. Spend 10 or 15 minutes reading Section 4.7 through the statement of Theorem 27. If you are interested in harmonic functions, you may find the Poisson integral formula (Theorem29) interesting. However, we will not be considering this section for our course. Then read Section 5.1 and the first part of the Chapter 5 notes on the website; hopefully you remember some of these results from calculus. It might be helpful to review a calculus book on some of these topics. One option is Chapter 3 of "Calculus Concentrate" on my website.
2. Work on exercises 1, 2, 7, 10, 11, and 14 in Section 5.1.
3. Turn in a solution for exercise 10.

for Wednesday, April 3

1. Look over Section 5.2 to get a sense of the results. However, it is not necessary to focus on all of the details, especially those appearing toward the end of the section.
2. Work on exercises 5, 7, 8, and 13 in Section 5.2. For problem 7, to verify the identity, note that both $1+z$ and $1-z$ lie in the right half plane. To find the series, use the series given in Example 1. Finally, use your answer (you can check your series with the answer in the back of the book) to find infinite series that converge to $\operatorname{Log} 2$ and $\operatorname{Log}(i/2)$. Here are a few additional problems.
   i. Find the Maclaurin series for the function $f(z) = \dfrac{1-\cos z}{z}$.
   ii. Determine (in our usual notation) the function represented by each of the following Maclaurin series.

$$\sum_{k=0}^{\infty} \frac{2^{2k}}{k!}\, z^k, \quad \sum_{k=0}^{\infty} \frac{(-1)^k 3^{2k}}{(2k+1)!}\, z^{2k+1}, \quad \sum_{k=0}^{\infty} \frac{(-5)^{k+1}}{(2k)!}\, z^{2k+1}, \quad \sum_{k=0}^{\infty} \frac{(1+i)^k}{k!}\, z^{2k}, \quad \sum_{k=0}^{\infty} \frac{(3+2i)^{2k}}{(2k+1)!}\, z^{2k+1}.$$

3. We will review for the second exam during this class period.

for Friday, April 5

1. We have our second test on this day, primarily covering the material in Chapters 3, 4, and 5 that we have discussed since the first exam. You should focus on the major concepts we have considered and be able to perform computations related to these concepts. Flipping through the sections we have covered and going over the problems that have been assigned should prepare you well for the exam.
2. To avoid the possible stress of time constraints, you may work on the exam until 12:15. If this creates a time conflict for you, please let me know.

for Monday, April 8

1. Review Sections 5.1 and 5.2 as needed. You should also look over the first six pages of the Chapter 5 notes on the website.
2. There is a special assignment due this coming Friday. The three problems (8 points each as usual and with the standard guidelines for working on these assignments) are Exercises 14 and 16 in Section 4.6 and Exercise 18 in Section 5.2. Note that a and b are complex numbers for Exercise 4.6.16; you need to "align" az^n and b to get equality.

for Wednesday, April 10

1. Read Section 5.3.
2. Work on Exercises 3, 4, 5, 7, 9, and 13. Note that 13c gets a little messy since you must use the series method.
3. Turn in solutions for exercises 4 and 13a. One of the examples in the Chapter 5 notes may prove helpful for 13a. Be sure to write the details carefully and clearly.

for Friday, April 12

1. You do not need to read Section 5.4. However, you should work on the following exercises as they provide good practice for this material. Use the ratio test for exercises 3a and 3d. Use some simple reasoning to answer exercises 5a, 5c, and 5d. Exercise 9 provides an illustration of the sorts of problems you can solve when treating functions like polynomials.
2. Read Section 5.5, focusing on the key results and the examples without letting the proofs overwhelm you.
3. Do exercises 1, 3, 4, 5, 6, and 9 in Section 5.5. Note that exercises 4 and 6 are very easy since we can use known series.
4. The fifth special assignment (see the 4/8 assignment) is due this day.

for Monday, April 15

1. Read Section 5.6. There are a lot of words here but the main ideas are straightforward.
2. Do exercises 1, 2, 3, 5, 6, and 12 in Section 5.6.
3. Turn in solutions for exercises 2 and 12.

for Wednesday, April 17

1. Read Section 5.7 and the portion of Section 5.8 through Corollary 5, focusing on getting an informal sense of these ideas.
2. Do exercises 1, 2, and 3 in Section 5.7 and exercises 1, 2, and 3 in Section 5.8.

for Friday, April 19

1. Read Section 6.1.
2. Do exercises 1a–f, 3abe, 4, 5, 6, and 7 in Section 6.1.
3. Turn in solutions for exercises 3e and 6 in Section 6.1. Ponder carefully various options for finding the residue for the function in 3e.

for Monday, April 22

1. Read Section 6.2.
2. Do exercises 1, 2, 4, and 5 in Section 6.2. For exercise 4, you might consider using a half-angle formula in a creative way.
3. You might find it interesting to show how exercises 1, 2, and 4 are actually special cases of exercise 5. In addition, with a little patience, you can show that exercise 8 follows from exercise 5 as well.
4. There is a special assignment due this day.

for Wednesday, April 24

1. Read Section 6.3 through the statement of Lemma 1 and Section 6.4 through Example 1.
2. Do exercise 3 in Section 6.3 and exercises 1 and 3 in Section 6.4.
3. For some calculus practice, do Exercise 6.3.1 using Calculus II ideas (there is a simple antiderivative) and do Exercise 6.3.2 by showing that it is a multiple of the integral in Example 2 (make a simple change of variables).
4. Turn in solutions for the shortcut method for Exercise 6.3.2 (see above item) and Exercise 3 in Section 6.4.

for Friday, April 26

1. Look over Section 6.5 to get a sense for how to do integrals of the type discussed there.
2. Do exercises 3, 4, 5, and 10 in Section 6.5. You might find that partial fractions help a little for exercise 3. Use the result of Example 2 and integration by parts for exercise 5; no complex theory is needed. Use the given identity and the result of exercise 5 to solve exercise 10 without using complex analysis.
3. Turn in solutions for exercises 4 and 5.

for Monday, April 29

1. Read Section 6.7; the Chapter 6 notes on the website may be helpful.
2. Do exercises 1, 2, 3, 6, 7, and 8 in Section 6.7.
3. Turn in a solution for exercise 8.

for Wednesday, May 1

1. We have our third test on this day, primarily covering the material in Chapters 5 and 6 that we have discussed since the second exam. You should focus on the major concepts we have considered and be able to perform computations related to these concepts. Flipping through the sections we have covered and going over the problems that have been assigned should prepare you well for the exam.
2. To avoid the possible stress of time constraints, you may work on the exam until 12:15. If this creates a time conflict for you, please let me know.

for Friday, May 3

1. The seventh special assignment is due this day.
2. We are skipping Section 7.1. Read Section 7.2 and read Section 7.3 through the solution to Example 2.

for Monday, May 6

1. Finish reading Section 7.3 and read Section 7.4 through the solution to Example 2.
2. Work on exercises 3cde, 5, 6, 7, and 12 in Section 7.3 and exercises 1, 2, and 8 in Section 7.4. Use techniques from Section 7.4 for exercise 7.3.12; try mapping $-i$, 1, ∞ to $-i$, 1, i respectively, then explain why this mapping has the desired property. Give the matrix method a try for solving Example 1 in Section 7.4.
3. We will talk about the final exam on this day.

4. The final exam is comprehensive, covering all of the sections in the textbook that have been assigned over the semester. To prepare for the final exam, you should go through the pages of the textbook to remind yourself of the key ideas, theorems, and problem types. You can look over the extra notes as a reminder of the results and examples that are covered there, and you can go over the assigned homework problems (including the special assignments) to revisit the methods used for solving various problems. You should able to identify about 30 key results; here is a sampling of some of these.

   Cauchy-Riemann equations

   Fundamental Theorem of Algebra

   ML Lemma

   Cauchy's Integral Formula

   Liouville's Theorem

   Maximum Modulus Theorem

   Cauchy Residue Theorem

   Argument Principle

   Rouché's Theorem

   Open Mapping Theorem

   You will not be asked to quote these theorems or results, but they may be helpful in solving test questions. (Recall the example we did in class on Monday using the Argument Principle.)