Remember that you are to work alone on this assignment with no help from other students/people or Internet resources. You may use the textbook and other course materials. You may ask me questions concerning clarifications of the problems, but I will not give hints in the same way that I do for regular homework. Please follow these guidelines so that cases of academic dishonesty do not arise.

This special assignment involves a little bit of practice with Fourier series on the interval $[-\pi, \pi]$. Recall that a continuous function f can be expressed as

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \bigl(a_n \cos(nt) + b_n \sin(nt)\bigr),$$

where

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos(nt)\, dt \qquad \text{and} \qquad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin(nt)\, dt.$$

(See the video for Section 6.8.) Write neat and concise solutions to the following problems. Include all of the relevant details and intermediate steps, with brief explanations as necessary. Pay particular attention to your writing; wording, use of notation, clarity, and neatness. As indicated above, you may use the textbook/videos and your notes, but no other sources, animate or inanimate. Part of your grade will be determined by the care with which you write your solutions. This assignment is worth 30 points and is due by noon (Pacific time) on Monday, May 3. The file should be named `email_sp_05.pdf`.

1. Find the Fourier series for the function $f(t) = t^2$.

2. Use the result of the previous problem to show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

3. Referring to the video for Section 6.8, start with the Fourier series for t^3 and integrate both sides (assuming that everything works fine even for the infinite sum) to obtain the Fourier series for t^4.

4. Use the result of the previous problem to show that $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^4} = \frac{\pi^4}{90}$.

5. Find the Fourier series for the function $f(t) = \begin{cases} 0, & \text{if } -\pi \le t < 0; \\ t, & \text{if } 0 \le t \le \pi. \end{cases}$