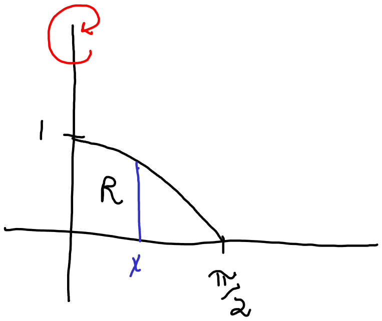
Name: due September 23 by 3:00 pm Pacific

Math 126 **Homework Assignment 11** **Fall 2020**

1. Let R be the region under the graph of $y = \cos x$ and above the x-axis on the interval $[0, \pi/2]$. Find the volume of the solid that is generated when R is revolved around the y-axis.



$\underline{\cos x\, dx}$ area

$\underline{x}$ radius of orbit

$\underline{2\pi x}$ distance traveled

$2\pi x \cos x\, dx$ tiny part of volume

$$V = \int_0^{\pi/2} 2\pi x \cos x\, dx$$

$u = x$ $\quad dv = \cos x\, dx$

$du = dx$ $\quad v = \sin x$

integration by parts

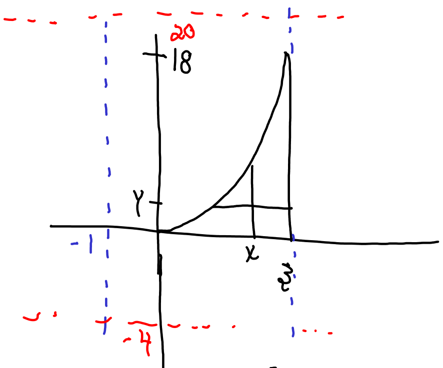
$$= 2\pi\left(x\sin x + \cos x\right)\Big|_0^{\pi/2}$$

$$\int x\cos x\, dx = x\sin x - \int \sin x\, dx$$
$$= x\sin x + \cos x + C$$

$$= 2\pi\left(\frac{\pi}{2} - 1\right) = \pi(\pi - 2)$$

The volume of the solid is $\pi^2 - 2\pi$ cubic units.

2. Let R be the region under the graph of $y = 2x^2$ and above the x-axis on the interval $[0, 3]$. Set up, but do not evaluate, an integral that represents the volume of the solid that is generated when R is revolved around (a) the line $x = 3$, (b) the line $y = 20$, (c) the line $x = -1$, and (d) the line $y = -4$.



$y = 2x^2$ $\qquad$ vertical $\quad 2x^2\, dx$

$x = \sqrt{\frac{y}{2}}$ $\qquad$ horizontal $\quad \left(3 - \sqrt{\frac{y}{2}}\right) dy$

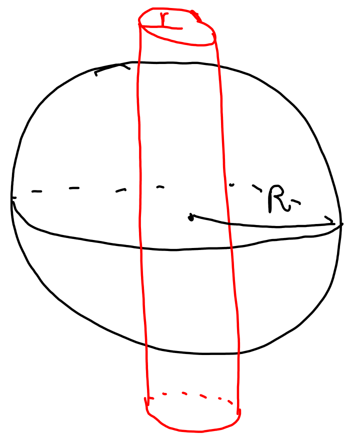
$$V_{x=3} = \int_0^3 2\pi(3 - x)\, 2x^2\, dx$$

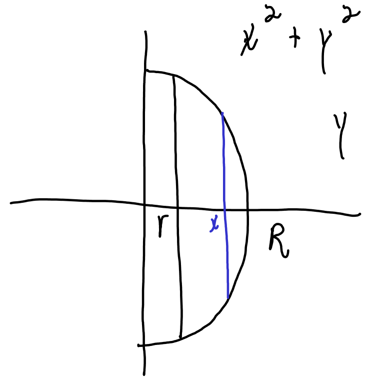
$$V_{x=-1} = \int_0^3 2\pi(x + 1)\, 2x^2\, dx$$

$$V_{y=20} = \int_0^{18} 2\pi(20 - y)\left(3 - \sqrt{y/2}\right) dy$$

$$V_{y=-4} = \int_0^{18} 2\pi(y + 4)\left(3 - \sqrt{y/2}\right) dy$$

3. A cylindrical hole of radius r is bored through the center of a sphere with radius $R > r$. The remaining solid resembles a bead since it has a flat top and bottom with a hole through the middle. Find the volume of the bead.





$$x^2 + y^2 = R^2$$

$$y = \pm\sqrt{R^2 - x^2}$$

$2\sqrt{R^2 - x^2}\,dx$ area

travels $2\pi x$

$$V = \int_r^R 2\pi x \, 2\sqrt{R^2 - x^2}\, dx$$

$$= -\frac{4\pi}{3}\left(R^2 - x^2\right)^{3/2}\Big|_r^R$$ [mental guess and check]

$$= \frac{4\pi}{3}\left(R^2 - r^2\right)^{3/2}$$ [note $r = 0$ gives expected result]

The volume of the bead is $\frac{4\pi}{3}\left(R^2 - r^2\right)^{3/2}$ cubic units.

If we want this to be half the volume of the sphere, then r must satisfy

$$\frac{4\pi}{3}\left(R^2 - r^2\right)^{3/2} = \frac{1}{2}\cdot\frac{4\pi}{3}R^3 \Rightarrow \left(R^2 - r^2\right)^{3/2} = \frac{1}{2}R^3$$

$$\Rightarrow R^2 - r^2 = \left(\frac{1}{2}\right)^{2/3} R^2$$

$$\Rightarrow r^2 = \left(1 - \frac{1}{\sqrt[3]{4}}\right) R^2$$

$$\Rightarrow r = \sqrt{1 - \frac{1}{\sqrt[3]{4}}}\; R$$

$$\approx 0.6083\, R$$