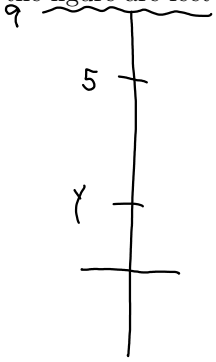
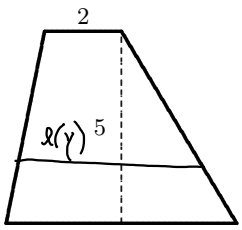
Name: due September 25 by 3:00 pm Pacific

Math 126 **Homework Assignment 12** **Fall 2020**

1. Find the force exerted by a liquid with weight density w on one side of the vertically submerged plate. The units on the figure are feet and the top of the plate is four feet beneath the surface of the liquid.





trapezoid

$l(0) = 6$, $l(5) = 2$
linear function
$l(y) = 6 - \frac{4}{5}y$

$$F = \int_0^5 w(9-y)\left(6 - \tfrac{4}{5}y\right)dy = \tfrac{2}{5}w\int_0^5 (9-y)(15-2y)\,dy$$

$$= \tfrac{2}{5}w\int_0^5 (2y^2 - 33y + 135)\,dy$$

$$= \tfrac{2}{5}w\left(\tfrac{2}{3}\cdot y^3 - \tfrac{33}{2}y^2 + 135y\right)dy$$

$$= \tfrac{2}{5}w\left(\tfrac{10}{3}\cdot 5^2 - \tfrac{33}{2}\cdot 5^2 + 27\cdot 5^2\right)$$

$$= 10w\left(\tfrac{25}{3} + \tfrac{1}{3} - 17 + \tfrac{1}{2} + 27\right)$$

$$= 10w\left(13 + \tfrac{5}{6}\right)$$

$$= 10w\cdot\tfrac{83}{6}$$

$$= 5w\cdot\tfrac{83}{3}$$

$$= \tfrac{415}{3}w$$

The force on the plate is $\frac{415}{3}w$ lbs.

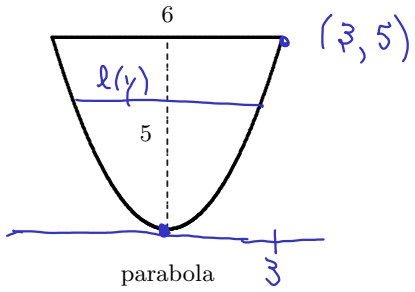
2. Find the force exerted by a liquid with weight density w on one side of the vertically submerged plate. The units on the figure are feet and the top of the plate is four feet beneath the surface of the liquid.

put origin at vertex

$y = kx^2$

$5 = k \cdot 9$

$k = \frac{5}{9}$



$y = \frac{5}{9}x^2$

$x = \pm \frac{3}{\sqrt{5}}\sqrt{y}$

$l(y) = \frac{6}{\sqrt{5}}\sqrt{y}$

$$F = \int_0^5 w(9-y)\frac{6}{\sqrt{5}}\sqrt{y}\,dy$$

$$= \frac{6w}{\sqrt{5}}\int_0^5 \left(9y^{1/2} - y^{3/2}\right)dy$$

$$= \frac{6w}{\sqrt{5}}\left(6y^{3/2} - \frac{2}{5}y^{5/2}\right)\Big|_0^5$$

$$= \frac{6w}{\sqrt{5}}\left(6 \cdot 5\sqrt{5} - \frac{2}{5}\cdot 25\sqrt{5}\right)$$

$$= 6w(30 - 10)$$

$$= 120w$$

$5^{3/2} = 5 \cdot 5^{1/2}$

$5^{5/2} = 5^2 \cdot 5^{1/2}$

The force exerted by the liquid is $120w$ pounds.