Name: due September 30 by 3:00 pm Pacific

Math 126 **Homework Assignment 14** **Fall 2020**

1. Find the length of the curve $y = \dfrac{2x^3}{3} + \dfrac{1}{8x}$ on the interval $[1, 2]$.

$$\frac{dy}{dx} = 2x^2 - \frac{1}{8x^2}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + 4x^4 - \frac{1}{2} + \frac{1}{64x^4} = 4x^4 + \frac{1}{2} + \frac{1}{64x^4} = \left(2x^2 + \frac{1}{8x^2}\right)^2$$

$$L = \int_1^2 \left(2x^2 + \frac{1}{8x^2}\right) dx = \left(\frac{2}{3}x^3 - \frac{1}{8x}\right)\Big|_1^2$$

$$= \frac{2}{3}(8-1) - \frac{1}{8}\left(\frac{1}{2} - 1\right) = \frac{14}{3} + \frac{1}{16} = \frac{227}{48}$$

The length of the curve is $\frac{227}{48}$ units.

2. Find the length of the curve $y = \ln|\cos x|$ on the interval $[0, \pi/3]$.

$$\frac{dy}{dx} = \frac{-\sin x}{\cos x} = -\tan x$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \tan^2 x = \sec^2 x$$ [basic trig identity]

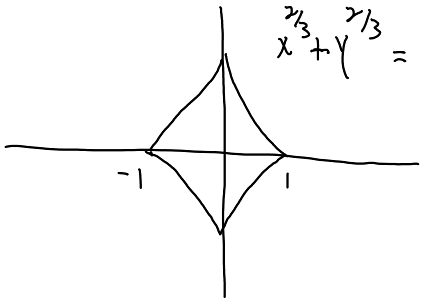
$$L = \int_0^{\pi/3} \sec x \, dx = \ln|\sec x + \tan x| \Big|_0^{\pi/3}$$ [see Appendix B integral 60]

$$= \ln|2 + \sqrt{3}| - \ln|1 + 0|$$

$$= \ln(2 + \sqrt{3}) \approx 1.317$$

The length of the curve is $\ln(2+\sqrt{3})$ units.

3. Find the length of the entire curve given by the equation $x^{2/3} + y^{2/3} = 1$. Include a sketch of the curve, noting that it has branches in all four quadrants.



find length in Q I, then multiply by 4

$$y^{2/3} = 1 - x^{2/3}$$

$$y = \pm\left(1 - x^{2/3}\right)^{3/2}$$ use plus sign for Q I

$$\frac{dy}{dx} = \frac{3}{2}\left(1 - x^{2/3}\right)^{1/2} \cdot \left(-\frac{2}{3}x^{-1/3}\right) = -\frac{\sqrt{1-x^{2/3}}}{x^{1/3}}$$

$$1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{1-x^{2/3}}{x^{2/3}} = \frac{1}{x^{2/3}}$$

$$L = 4\int_0^1 x^{-1/3}\,dx = 4 \cdot \frac{3}{2}x^{2/3}\Big|_0^1 = 6$$

{ technically, this is an improper integral since $\frac{1}{x^{1/3}}$ is unbounded on $[0,1]$ }

The length of the entire curve is 6 units.