Name: due October 23 by 3:00 pm Pacific

**Math 126** **Homework Assignment 21** **Fall 2020**

1. Prove that the series $\sum_{k=1}^{\infty} \frac{7}{\sqrt[k]{4}+3}$ diverges.

Note that $\lim_{k\to\infty} \frac{7}{\sqrt[k]{4}+3} = \frac{7}{1+3} = \frac{7}{4}$. Since the sequence $\left\{\frac{7}{\sqrt[k]{4}+3}\right\}$ does not converge to 0, the series $\sum_{k=1}^{\infty} \frac{7}{\sqrt[k]{4}+3}$ diverges by the Divergence Test.

2. Find the sum of the series $\sum_{k=1}^{\infty} \frac{(-1)^k 2^{k+1}}{3^k}$.

This is a geometric series with $r = -\frac{2}{3}$ and $a = -\frac{4}{3}$.

It follows that

$$\sum_{k=1}^{\infty} \frac{(-1)^k 2^{k+1}}{3^k} = \frac{-4/3}{1-\left(-\frac{2}{3}\right)} = -\frac{4/3}{5/3} = -\frac{4}{5}.$$

The series converges since $|r| < 1$.

We find a by using $k = 1$ to get the first term.

$$\sum_{k=1}^{\infty} \frac{(-1)^k 2^{k+1}}{3^k} = \sum_{k=1}^{\infty} \frac{(-1)^k 2^k \cdot 2}{3^k} = \sum_{k=1}^{\infty} 2\left(-\frac{2}{3}\right)^k.$$

3. Find the sum of the series $\sum_{k=2}^{\infty} \frac{5k^2-3}{2^k}$ given that $\sum_{k=1}^{\infty} \frac{k^2}{2^k} = 6$. Pay attention to the starting value of the index; it is different for each series.

Since $6 = \sum_{k=1}^{\infty} \frac{k^2}{2^k} = \frac{1}{2} + \sum_{k=2}^{\infty} \frac{k^2}{2^k}$, it follows that $\sum_{k=2}^{\infty} \frac{k^2}{2^k} = \frac{11}{2}$. By properties of series, we have

$$\sum_{k=2}^{\infty} \frac{5k^2-3}{2^k} = 5 \sum_{k=2}^{\infty} \frac{k^2}{2^k} - \sum_{k=2}^{\infty} \frac{3}{2^k}$$

$$= 5 \cdot \frac{11}{2} - \frac{3/4}{1-\frac{1}{2}} \qquad \text{(geometric series)}$$

$$= \frac{55}{2} - \frac{3}{2}$$

$$= 26.$$

The sum of the series is 26.

4. Let $\sum_{k=1}^{\infty} a_k$ be an infinite series and suppose that $s_n = \frac{2n+5}{3n-4}$ for all $n \geq 1$, where $\{s_n\}$ represents the corresponding sequence of partial sums. Find $a_1$, $a_2$, $a_{10}$, and the sum of the series.

By the definition, we have

$$\sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} \sum_{k=1}^{n} a_k = \lim_{n\to\infty} s_n = \lim_{n\to\infty} \frac{2n+5}{3n-4} = \frac{2}{3}.$$

This gives the sum of the series. Using the fact that $s_n = \sum_{k=1}^{n} a_k$, we find that

$a_1 = s_1 = \frac{7}{-1} = -7$;

$a_2 = s_2 - s_1 = \frac{9}{2} - (-7) = \frac{23}{2}$;

$a_{10} = s_{10} - s_9 = \frac{25}{26} - \frac{23}{23} = -\frac{1}{26}$.