Name: due May 6

Math 125 **Homework Assignment 31** **Spring 2022**

1. Evaluate $\displaystyle\int \frac{24}{(4x+1)^2}\,dx$.

2. Evaluate $\displaystyle\int 9\sqrt[3]{2x+7}\,dx$.

3. Evaluate $\displaystyle\int_0^1 \frac{2x+2}{2x^2+4x+1}\,dx$.

4. Evaluate $\displaystyle\int_0^3 (6x-8)\sqrt{9-x^2}\,dx$. (Carefully use the distributive property to split the integral into two integrals, then think carefully about the best way to evaluate each of the integrals.)

Name: due May 4

Math 125 **Homework Assignment 30** **Spring 2022**

1. Evaluate $\displaystyle\int_1^3 \frac{18}{x^2}\,dx$.

2. Evaluate $\displaystyle\int_0^1 \left(2x^3 - \sqrt[4]{x}\right) dx$.

3. Evaluate $\displaystyle\int_0^3 \frac{4}{2x+3}\,dx$.

4. Evaluate $\int_{-3}^{3} 12\sqrt{9-x^2}\,dx$. (Please think first.)

5. Find the area of the region under the curve $y = 6/(1+x^2)$ and above the x-axis on the interval $[-1, 1]$.

6. Evaluate $\int_{0}^{1} 6e^{-2x}\,dx$.

Name: due May 2

Math 125 **Homework Assignment 29** **Spring 2022**

1. Find the derivative of the function f defined by $f(x) = \displaystyle\int_0^{2x^2} \sqrt[3]{5t^2 + t}\, dt$.

2. Determine $F''(5)$ given that $F(x) = \displaystyle\int_x^{12} f(t)\, dt$ and $f(x) = \displaystyle\int_1^{4x} \frac{\ln(1+t)}{t}\, dt$.

3. Evaluate $\lim_{x \to 0} \frac{1}{x^5} \int_0^x \left(1 - \cos(t^2)\right)\, dt$.

4. Find an integral expression for a function f such that $f(1) = 0$ and $f'(x) = x^2 e^{-x^2}$.

Name: due April 29

Math 125 **Homework Assignment 28** **Spring 2022**

1. Use simple facts from geometry to find the area under the graph of each function and above the x-axis on the given interval. Include a sketch of the region whose area is being computed.

   a) $f(x) = 8 - |x - 2|$ on $[0, 6]$

   b) $g(x) = \sqrt{10x - x^2}$ on $[0, 10]$

2. Find the area under the curve $y = x^2$ and above the x-axis on the interval $[3, 6]$.

3. Use the definition of the integral to express the given integral as a limit of a sum.

a) $\displaystyle\int_1^5 (x^2 + 4x)\, dx$

b) $\displaystyle\int_0^{\pi/3} \cos x\, dx$

4. Use the definition of the integral to express the given limit as an integral. For part (a), give two different options for the integral; one with 0 as the lower limit of integration and one with 4 as the lower limit of integration. For part (b), try some factoring as a first step to make $\dfrac{1}{n}$ appear.

a) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{4 + \frac{3i}{n}} \cdot \frac{3}{n}$

b) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n}{n^2 + i^2}$

Name: due April 27

Math 125 **Homework Assignment 27** **Spring 2022**

1. Express the sum $\frac{2}{3} + \frac{4}{5} + \frac{6}{7} + \cdots + \frac{40}{41}$ using summation notation.

2. Find the sum $\sum_{k=11}^{20} k^3$. See if you can do this without a calculator.

3. Evaluate $\displaystyle\lim_{n\to\infty} \frac{4n^3}{1^2+2^2+3^2+\cdots+n^2}$.

4. Evaluate $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+3}\right)$. First find the sum as a function of n; see Exercise 2.1.4b.

Name: due April 20

Math 125 **Homework Assignment 26** **Spring 2022**

1. A lighthouse is one kilometer away from a straight shoreline and its light makes four revolutions per minute. How fast is the beam of light moving along the shoreline when the light is two hundred fifty meters from the nearest point on the shoreline?

2. Sand is leaking out of an elevated bin and falling into a conical pile at the rate of two cubic feet per minute. The radius of the base of the cone is always three times the height of the cone. At what rate is the circumference of the base increasing when the height of the pile is four feet?

Name: due April 18

Math 125 **Homework Assignment 25** **Spring 2022**

1. Use implicit differentiation to find dy/dx given $x^3 + y^3 = 30xy$.

2. Use implicit differentiation to find dy/dx given $x^3 + 4xy^3 + y^4 = 150$.

3. Find an equation for the line tangent to the curve $y^4 - x^2y + 4x - 3y - 1 = 0$ at the point $(3, 1)$.

4. Find an equation for a tangent line to the curve $x^2 - y^2 = 5$ that goes through the point $(1, 1)$. (Note that the point $(1, 1)$ is not on the curve.)

Name: due April 15

Math 125 **Homework Assignment 24** **Spring 2022**

1. Find the position function $s(t)$ given that $a(t) = 6t + 4$, $v(1) = 10$, $s(0) = 30$.

2. Suppose that an object is moving in a straight line with a velocity of 120 ft/sec. At time $t = 0$, the object begins to decelerate at the rate of $5\sqrt{t}$ ft/sec^2. How far does this object travel during the braking process? Give both an exact answer for this distance and an approximate answer to the nearest foot.

3. A hot air balloon is ascending at the rate of 20 m/sec at a height 100 m above the ground when a brick is dropped from the balloon. How long does it take for the brick to reach the ground? With what speed does it hit the ground? Note that the initial velocity of the brick is not 0. (Have you ever stepped out of a moving car?) You may use a calculator for the necessary computations.

Name: due April 13

Math 125 **Homework Assignment 23** **Spring 2022**

1. Find and simplify the second derivative of the function $f(x) = \dfrac{\ln x}{x^2}$.

2. For the function $g(x) = 3x^2 - x^3$, determine the intervals on which g is concave up and those on which g is concave down. Also, find the (x, y) coordinates of any inflection points.

3. For the function $f(x) = xe^{-x/4}$, determine the intervals on which f is concave up and those on which f is concave down. Also, find the (x, y) coordinates of any inflection points.

4. Find a polynomial P of degree 3 so that P has an inflection point at $(1, 6)$ and a y-intercept of 4.

Name: due April 11

Math 125 **Homework Assignment 22** **Spring 2022**

1. Find the point c (give the exact value of this point) guaranteed by the Mean Value Theorem for the function $f(x) = x^3 - 15x^2 + 12x$ on the interval $[1, 4]$.

2. Find the point c (give the exact value of this point) guaranteed by the MVT for the function $f(x) = \dfrac{18}{x^2}$ on the interval $[1, 3]$.

3. Prove that the function $f(x) = 7x + 2\sin(3x)$ is increasing on $\mathbb{R}$.

4. Find the most general form for $g(x)$ given that $g'(x) = x^3 + 2x - 7$.

5. Find the most general form for $h(t)$ given that $h'(t) = \dfrac{12t^3}{2 + t^4}$.

Name: due April 8

Math 125 **Homework Assignment 21** **Spring 2022**

1. Evaluate $\lim_{x\to 1} \dfrac{3x^5+4x^4-2x-5}{6x-x^3-5x^5}$. Show all of your steps and pay careful attention to notation.

2. Evaluate $\lim_{x\to 0} \dfrac{e^{2x}-1}{\sin(3x)+2x}$. Show all of your steps and pay careful attention to notation.

3. Evaluate $\lim_{x\to 0} \dfrac{x^2}{\cos x-\cos(2x)}$. Show all of your steps and pay careful attention to notation.

4. Evaluate $\lim_{x \to 0} \left(e^x + \sin(3x)\right)^{1/x}$. Show all of your steps and pay careful attention to notation.

5. Evaluate $\lim_{x \to \infty} \left(xe^{-3/x} - x\right)$.

Name: due April 6

Math 125 **Homework Assignment 20** **Spring 2022**

1. Evaluate $\lim_{x\to\infty} \frac{\sqrt{2x^2+3x+20}}{4x+7}$. Show all of your steps and pay careful attention to notation.

2. Evaluate $\lim_{x\to\infty} \frac{3x^5+4x^4-2x-5}{6x-x^3-5x^5}$. Show all of your steps and pay careful attention to notation.

3. Evaluate $\lim_{x\to\infty} \left(x-\sqrt{x^2+x+2}\right)$. Show all of your steps and pay careful attention to notation.

4. Evaluate $\lim_{x \to 3^-} \dfrac{x^4}{x^2 - 9}$. Explain how you arrived at your answer.

5. Find all of the asymptotes, both vertical and horizontal, for the function $g(x) = \dfrac{x^2 + 5x + 4}{3x^2 + x - 2}$. Be careful and be certain to explain your answers.

6. Find all of the vertical asymptotes for the function $g(x) = \dfrac{\ln x}{x - 2}$. Be careful and be certain to explain your answers.

Name: due April 4

Math 125 **Homework Assignment 19** **Spring 2022**

1. An island in a lake is located 600 yards opposite one end of a portion of the straight shoreline that is 3 miles long. Suppose you can swim at a rate of 3.2 miles per hour and can run at a rate of 9.5 miles per hour. Ignoring transition time, find the least amount of time required to get to the other end of the shoreline from the island. Give your answer to the nearest second. Be certain to check the endpoints that arise in this problem since they both give reasonable options for a race. (You will need a calculator for some of the computations.)

Name: due April 1 (no kidding)

Math 125 **Homework Assignment 18** **Spring 2022**

1. Find and simplify the derivative of $f(x) = 3e^{-x/3}\cos(2x)$.

2. Find and simplify the derivative of $g(t) = \ln(t^2 + 6t + 10)$.

3. Find the (x, y) coordinates (in exact form) of the lowest point on the graph of $y = 16e^{-x} + e^{2x}$.

4. Determine the intervals on which f is increasing and those on which it is decreasing for $f(x) = \dfrac{x^4}{\ln x}$.

5. Find the (x, y) coordinates (in exact form) of the point on the graph of $y = 3e^{x/2}$ for which the tangent line to the curve passes through the origin.

Name: due March 30

Math 125 **Homework Assignment 17** **Spring 2022**

1. Suppose that $\log_a 2 = r$, $\log_a 3 = s$, and $\log_a 7 = t$, where a is some positive number. Express $\log_a 21$, $\log_a(7/4)$, $\log_a(126)$, $\log_a(3/98)$, and $\log_a \sqrt[3]{6}$ in terms of r, s, and t.

2. Find the exact value of x that satisfies the equation $\ln(5x+3) - \ln 4 = 2$.

3. Find the exact value of x that satisfies the equation $\dfrac{8e^{-x/2}}{3 + e^{-x/2}} = 1$, then use a calculator to approximate x to the nearest thousandth.

4. Find the exact value of x that satisfies the equation $e^x - 6e^{-x} = 1$.

5. The college debt of a student at time t years is given by $D(t) = 10000\left(3 - e^{0.06t}\right)$ dollars. How much is the original debt? What is the debt after 10 years? When will the debt be \$1000?
(You may use a calculator to find these values after expressing them in exact form.)

Name: due March 28

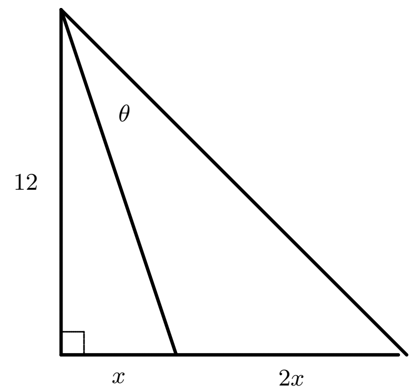
Math 125 **Homework Assignment 16** **Spring 2022**

1. Find and simplify the derivative of $f(x) = \arctan(x/3)$.

2. Find and simplify the derivative of $r(\theta) = \theta \arcsin \theta + \sqrt{1 - \theta^2}$.

3. Find the maximum and minimum outputs of the function $f(x) = \sin^2 x + \cos x$ on the interval $[0, \pi]$.

4. Referring to the given figure, find the value of $x \in (0, \infty)$ that will maximize θ. Start by writing θ as the difference of two angles using the arctan function.



Name: due March 7

Math 125 **Homework Assignment 15** **Spring 2022**

1. Find and simplify the derivative of the function f defined by $f(x) = 2\cos^3(5x)$.

2. Find and simplify the derivative of the function g defined by $g(x) = x\tan(\pi x)$.

3. Find and simplify (think carefully) the derivative of the function h defined by $h(x) = \cos x - \frac{1}{3}\cos^3 x$.

4. Find all values of x in the interval $[0, 3\pi]$ for which the tangent line to the graph of $y = x + 2\cos x$ is horizontal.

5. Find the maximum and minimum outputs of the function $f(x) = \dfrac{\sin x}{2 + \cos x}$ on the interval $[0, \pi]$.

Name: due March 2

Math 125 **Homework Assignment 14** **Spring 2022**

1. Find the maximum possible area of a rectangle with base that lies on the x-axis and with two upper vertices on the graph of the circle $x^2 + y^2 = r^2$. Treat r as a positive constant. What percentage of the semicircle is occupied by this largest rectangle?

2. A rectangular box with a square base is to be constructed using materials that cost six dollars per square foot for the sides, ten dollars per square foot for the base, and five dollars per square foot for the top. Assuming that the volume of the box must be twenty cubic feet, determine the minimum cost of construction. Give your answer to the nearest dollar.

Name: due February 28

Math 125 **Homework Assignment 13** **Spring 2022**

1. Determine the nature of all of the critical points for $f(x) = x^3 + 3x^2 - 24x + 7$.

2. Determine the nature of all of the critical points for $f(x) = x^2 - \dfrac{16}{x}$.

3. Find the minimum distance from a point on the curve $y = \dfrac{12}{\sqrt{x}}$ to the origin.

Name: due February 25

Math 125 **Homework Assignment 12** **Spring 2022**

1. Determine the intervals on which f is increasing and those on which it is decreasing for $f(x) = 60x^2 - x^4$.

2. Determine the intervals on which f is increasing and those on which it is decreasing for $f(x) = x^3 + \dfrac{3a^4}{x}$, where a is a positive constant.

Name: due February 23

Math 125 **Homework Assignment 11** **Spring 2022**

1. Find the maximum and minimum outputs of the function $f(x) = 2x^3 - 3x^2 - 12x + 40$ on interval $[-2, 3]$.

2. Find the maximum and minimum outputs of the function $g(x) = \dfrac{10x}{x^2 + 10}$ on interval $[-2, 5]$.

3. The sum of two nonnegative numbers is 200. Find such numbers so that the product of the square of one with the cube of the other is as large as possible.

Name: due February 14

Math 125 **Homework Assignment 10** **Spring 2022**

1. Find and simplify the derivative of the function $f(x) = \dfrac{3x+1}{x^2+4}$.

2. Find and simplify the derivative of the function $g(t) = t\sqrt{12-t^2}$.

3. Determine all the values of x for which the tangent line to the graph of $y = (x-2)^3(3x+1)^4$ is horizontal.

4. The position s in meters of a particle at time t seconds is given by $s = 10t/(2t+1)$. Find the time $t > 0$ when the velocity of the particle 0.1 meters per second.

Name: due February 9

Math 125 **Homework Assignment 9** **Spring 2022**

1. Find and simplify the derivative of the function $f(x) = \dfrac{3}{x} - \dfrac{7}{3x^4}$.

2. Find and simplify the derivative of the function $g(t) = \dfrac{t^4 - \sqrt{t}}{t^2}$. (Do some algebra first.)

3. Find an equation for the line tangent to the graph of $f(x) = 3x^2 - \frac{8}{x}$ when $x = 2$.

4. Find all of the points on the curve $y = \sqrt{x}$ for which the tangent line goes through the point $(-16, 3)$.

Name: due February 7

Math 125 **Homework Assignment 8** **Spring 2022**

1. Find the derivative of the function $f(x) = \frac{3}{4}x^4 + \frac{2}{3}x^3 - \frac{1}{2}x^2 + 11$.

2. Consider the function $g(x) = x^5 - 3x^3 + 2x^2 + 4x$. Find equations for both the tangent line and the normal of this function when $x = 1$.

3. Suppose that the position of a particle is given by $s(t) = 20t^2 - t^3$, where t is measured in seconds and $s(t)$ in meters. Find the velocity of the particle when $t = 5$ seconds.

4. There are two points on the graph of $y = x^2 + 10x + 100$ for which the tangent line to the curve passes through the origin. Find the (x, y) coordinates of these two points.

Name: due February 4

Math 125 **Homework Assignment 7** **Spring 2022**

1. Use the definition of the derivative to find the derivative of the function $f(x) = 2x^2 - 5x + 7$.

2. Use the definition of the derivative to find the derivative of the function $g(x) = \sqrt{x}$.

3. Consider the function $f(x) = 27 - 3x^2$ and let ℓ be the tangent line to the curve $y = f(x)$ when $x = 1$. The line ℓ cuts off a triangle in the first quadrant; find the area of this triangle. (You may use the fact that $f'(x) = -6x$.)

4. Give two examples of a function f such that f is continuous for all values of x and differentiable for all values of x except $x = -2$ and $x = \frac{7}{3}$. One example should be given using a formula and another (different) example using just a careful sketch.

Name: due February 2

Math 125 **Homework Assignment 6** **Spring 2022**

1. Find the slope of the curve $y = f(x)$ at $x = 2$ when $f(x) = x^2 + 3x$.

2. Find the slope of the curve $y = f(x)$ at $x = 9$ when $f(x) = 2\sqrt{x}$.

3. Find the slope of the curve $y = f(x)$ at a generic point $c \neq 0$ when $f(x) = \dfrac{4}{x}$.

4. There are two points on the curve $y = 4/x$ for which the tangent line to the curve goes through the point $(3, 1)$. Find these two points. (We know the slope from the previous problem; you should have obtained $-4/c^2$. See the extra notes for an example of what to do next.)

Name: due January 31

Math 125 **Homework Assignment 5** **Spring 2022**

1. Evaluate $\lim\limits_{x \to 3} \dfrac{2x^2 - 7x + 3}{x^2 + x - 12}$. Show your steps clearly.

2. Evaluate $\lim\limits_{x \to 2} \dfrac{x^4 - 16}{x^2 - 2x}$. Show your steps clearly.

3. Evaluate $\lim_{x \to 25} \dfrac{\sqrt{x} - 5}{x - 25}$. Show your steps clearly.

4. Evaluate $\lim_{x \to 0} \dfrac{x}{\tan(7x)}$. Show your steps clearly.

Name: due January 28

Math 125 **Homework Assignment 4** **Spring 2022**

1. Give an example of a rational function f such that f has a degree two polynomial in its denominator and f is continuous for all real numbers.

2. List the discontinuities of the function $g(x) = \dfrac{3x+4}{x^4 - 3x^2 - 10}$.

3. List the discontinuities of the function $h(x) = \sec x$ that are in the interval $[0, 3\pi]$.

4. Prove (note the word prove) that the equation $2 \sin x = x$ has a positive solution.

5. Consider the function f defined by $f(x) = \begin{cases} 4 - x^2, & \text{if } x \leq 2; \\ x + 2, & \text{if } x > 2. \end{cases}$ Sketch a careful graph of this function, then show that f does not satisfy the conclusion of the Intermediate Value Theorem on the interval $[1, 5]$. (Remember to check every value of v between $f(1)$ and $f(5)$.)

Name: due January 26

Math 125 **Homework Assignment 3** **Spring 2022**

1. Use algebra to evaluate $\lim_{x\to 2} \dfrac{x^2+3x-10}{3x^2-5x-2}$.

2. Use a calculator to estimate $\lim_{x\to 0} \dfrac{10^x-1}{x}$ to the nearest thousandth. Include a table of values with both positive and negative inputs.

3. Evaluate $\lim_{x\to 6^+} \big(2x - \lfloor x \rfloor\big)$.

4. Evaluate $\lim_{x\to -6^-} \big(\lfloor x/2 \rfloor - \lfloor x \rfloor\big)$.

5. Explain why the limit $\lim_{x\to 3} \dfrac{x^2 - 3x}{|x-3|}$ does not exist.

Name: due January 24

Math 125 **Homework Assignment 2** **Spring 2022**

1. Find the domain of the function $f(x) = \sqrt{9 - x^2}$. Express your answer in interval notation.

2. Find the domain of the function $g(x) = \dfrac{x+2}{x^2 - x + 56}$.

3. Give an example of a rational function $F(x)$ for which $F(2) = 0$ and F is undefined at ± 3.

4. Consider the functions $f(x) = x^2 - 2x$ and $g(x) = x + 3$. Find and simplify each of the following functions: $(f \circ f)(x)$, $(f \circ g)(x)$, $(g \circ f)(x)$, and $(g \circ g)(x)$. Note that these functions can also be written as $f(f(x))$, $f(g(x))$, $g(f(x))$, and $g(g(x))$, respectively.

5. Consider the function $h(x) = x^2 + x$. Find and fully simplify the quantity $\dfrac{h(x+t) - h(x-t)}{2t}$.

Name: due January 21

Math 125 **Homework Assignment 1** **Spring 2022**

1. Find an equation for the line that goes through the points $(-2, 3)$ and $(4, 7)$.

2. Find the point of intersection of the lines $y = 2x + 3$ and $y + 3x = 17$.

3. Let ℓ be the line that is the perpendicular bisector of the line segment joining $(1, 1)$ and $(7, 11)$. The line ℓ cuts off a triangle in the first quadrant. Find the area of this triangle.