Name: due May 6

Math 126 **Homework Assignment 34** **Spring 2024**

1. A five hundred gallon tank contains four hundred gallons of brine (salt dissolved in water) with a concentration of one-fourth pound of salt per gallon of water. A brine containing one pound of salt per gallon of water runs into the tank at the rate of five gallons per minute, and the well-stirred mixture runs out of the tank at the same rate. When will there be two hundred pounds of salt in the tank?

2. Suppose that you have acquired a college debt of \$24000. The annual interest rate is 4% and you pay \$200 each month. How long does it take you to pay off your loan and how much do you pay altogether? (See problem 9 in Section 1.22 for a hint, assuming continuous compounding and payments.)

Name: due May 3

Math 126 **Homework Assignment 33** **Spring 2024**

1. Find a function f such that $f'(x) = \dfrac{4}{2x+1}$ and $f(0) = 8$.

2. Find a function g such that $g'(x) = \dfrac{8}{x^2+4}$ and $g(2) = 11$.

3. Find a function h such that $h'(x) = 12\sin(4x)$ and $h(\pi/4) = 17$.

4. Find a function f such that $f'(x) = -2f(x)$ and $f(0) = 60$.

5. Find a function g such that $g'(x) = 3g(x)$ and $g(1) = 10$.

6. Find a function h such that $h'(t) = 40 - \frac{1}{3}h(t)$ and $h(0) = 15$.

Name: due May 1

Math 126 **Homework Assignment 32** **Spring 2024**

1. Evaluate $\displaystyle\int \frac{2x+3}{x^2-3x-4}\,dx$.

2. Evaluate $\displaystyle\int \frac{x^2 - 2x + 7}{x^3 + 4x}\, dx$.

Name: due April 24

Math 126 **Homework Assignment 31** **Spring 2024**

1. Use the Maclaurin series for $\sin x$ to find the Maclaurin series for the function $\dfrac{\sin x - x}{x^3}$.

2. Use the Maclaurin series for $\sin x$ to find the Maclaurin series for the function $\displaystyle\int_0^x \frac{\sin t}{t}\,dt$.

3. Use known Maclaurin series to represent the series $\sum_{k=0}^{\infty} \frac{(-1)^k}{k!} x^{2k+1}$ as a more familiar function.

4. For the function $f(x) = xe^{x^3}$, find $f^{(61)}(0)$. (First find the Maclaurin series for f, then look again at Theorem 3.23.)

Name: due April 22

Math 126 **Homework Assignment 30** **Spring 2024**

1. Use a geometric series to find a power series expression centered at 1 for the function $f(x) = \dfrac{2}{7-4x}$ and determine the interval of convergence for the resulting series. (Note that finding the interval of convergence should be very easy and require minimal effort.)

2. Use results in Section 3.11 (as well as some basic ideas) to find the sum of the series $\displaystyle\sum_{k=1}^{\infty} \frac{3k^2+k+1}{4^k}$.

3. Starting with the solution to problem 2 in Section 3.11 (it can be found in the back of the book) and taking derivatives, find a formula for the sum of the series $\sum_{k=1}^{\infty} k^3 x^k$. Use your formula to find $\sum_{k=1}^{\infty} \frac{k^3}{3^k}$.

Name: due April 19

Math 126 **Homework Assignment 29** **Spring 2024**

1. Find the interval of convergence for the power series $\sum_{k=0}^{\infty} \frac{4}{(k+2)3^k}(x-1)^k$. Be certain to check whether or not the series converges at the endpoints.

2. Give an example of a power series that has $[2, 8)$ as its interval of convergence.

3. Referring to Exercise 7 in Section 3.10, find a simple expression for the function represented by the power series $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{4^{k+1}} (x-3)^k$. In addition, determine both the radius of convergence and the interval of convergence for this series, noting that you should NOT need to use the Root or Ratio Test to do so.

Name: due April 17

Math 126 **Homework Assignment 28** **Spring 2024**

1. Determine whether or not the series $\displaystyle\sum_{k=1}^{\infty}\left(\frac{2k}{3k+\sqrt[k]{k}}\right)^k$ converges.

2. Determine whether or not the series $\displaystyle\sum_{k=1}^{\infty}\frac{(-2)^k k!}{4\cdot 7\cdot 10\cdot\cdots\cdot(3k+1)}$ converges.

3. Determine if the series $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k^3}{3^k}$ converges absolutely, converges conditionally, or diverges.

4. Find all values of x for which the series $\displaystyle\sum_{k=1}^{\infty}(-1)^{k+1}\frac{k}{4^k}(x-1)^k$ converges absolutely. *Hint*: Express ℓ as a function of x.

Name: due April 15

Math 126 **Homework Assignment 27** **Spring 2024**

1. Show that the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k^2}{2k^4+13}$ is absolutely convergent.

2. Determine (with proof and/or explanation) if the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k}(k+1)}{2k^3+7k^2-1}$ is absolutely convergent, conditionally convergent, or divergent.

3. Give an example of a series for which $\sum_{k=1}^{\infty} a_k$ converges but $\sum_{k=1}^{\infty} a_k^2$ diverges. (Note that the result of Exercise 5 in the textbook is relevant here.)

4. Carefully prove that the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}k}{3k^2+2}$ is conditionally convergent. (Note that two proofs are required; one to show a series diverges and another to show a series converges.)

Name: due April 12

Math 126 **Homework Assignment 26** **Spring 2024**

1. Use the Comparison Test to determine whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{7}{2k+3\sqrt{k}}$ converges.

2. Use the Limit Comparison Test to determine whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{k^2+5}{3k^4+2k^3-4}$ converges.

3. Determine (with proof) whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{k-4}{k^2-3k+7}$ converges.

4. Determine (with proof) whether or not the series $\displaystyle\sum_{k=1}^{\infty} \frac{4^k}{2k+5^k}$ converges.

Name: due April 10

Math 126 **Homework Assignment 25** **Spring 2024**

1. Use the Integral Test to show that $\sum_{k=2}^{\infty} \frac{1}{k \ln k}$ diverges and $\sum_{k=2}^{\infty} \frac{1}{k(\ln k)^2}$ converges. Use correct notation for improper integrals and note that the lower limit of integration is 2.

2. Find all values of a, where a is a real number, for which the series $\sum_{k=1}^{\infty} \frac{1}{k^{6a-a^2}}$ converges.

3. For each positive integer n, let $z_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} - \int_1^n \frac{dx}{x}$. Use the ideas in this section to prove that $\{z_n\}$ is a decreasing sequence of positive terms and thus convergent. For the decreasing part, it is best to consider $z_n - z_{n+1}$ since the terms of the sequence involve additions. To show that all of the terms are positive, look carefully at the inequalities next to the graph in the section.

Name: due April 8

Math 126 **Homework Assignment 24** **Spring 2024**

1. Prove that the series $\displaystyle\sum_{k=1}^{\infty} \frac{7}{\sqrt[k]{4}+3}$ diverges.

2. Find the sum of the series $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^k 2^{k+1}}{3^k}$.

3. Find the sum of the series $\sum_{k=2}^{\infty} \frac{5k^2 - 3}{2^k}$ given that $\sum_{k=1}^{\infty} \frac{k^2}{2^k} = 6$. Pay attention to the starting value of the index; it is different for each of these two series.

4. Let $\sum_{k=1}^{\infty} a_k$ be an infinite series and suppose that $s_n = \frac{2n+5}{3n-4}$ for all $n \geq 1$, where $\{s_n\}$ represents the corresponding sequence of partial sums. Find a_1, a_2, a_{10}, and the sum of the series.

Name: due April 5

Math 126 **Homework Assignment 23** **Spring 2024**

1. For each positive integer n, let $x_n = \frac{1}{n+1} + \frac{1}{n+2} + \frac{1}{n+3} + \cdots + \frac{1}{2n}$. Prove that the sequence $\{x_n\}$ converges. (To prove that the sequence is bounded, consider using some over and under estimates for the terms in the sum. As a start, you should write out the first four terms of the sequence, that is, determine x_1, x_2, x_3, and x_4. For monotone, consider the expression $x_{n+1} - x_n$.)

2. Let $r_1 = 6$ and $r_{n+1} = \frac{r_n}{2} + \frac{7}{r_n}$ for each $n \geq 1$. Suppose that we already know that the sequence $\{r_n\}$ converges. Under this assumption, find the limit of the sequence.

3. Define a sequence $\{a_n\}$ by $a_1 = 1$ and $a_{n+1} = 3 - (1/a_n)$ for $n \geq 1$. We have already proved that $1 \leq a_n \leq 3$ for all n (see Exercise 3.1.5). Using similar ideas, use math induction to prove that $\{a_n\}$ is an increasing sequence. Then conclude that $\{a_n\}$ converges and find its limit.

Name: due April 3

Math 126 **Homework Assignment 22** **Spring 2024**

1. Turn in a solution to problem 1b in Section 3.3. This solution should be formatted almost exactly like the first example in the last paragraph just prior to the exercises. For the record, there should be four clear steps and thus four references to theorems.

2. Find the limit of the sequence $\left\{\left(1-\dfrac{1}{3n}\right)^n\right\}$. Explain your reasoning.

3. Turn in a solution to problem 2f in Section 3.3. Use the squeeze theorem and use estimates for sums like some of the examples you have seen, either in class lectures or in the extra notes.

4. Turn in a solution to problem 4 in Section 3.3. Once again, use the squeeze theorem.

Name: due April 1

Math 126 **Homework Assignment 21** **Spring 2024**

1. Find the limit of the sequence $\left\{\sqrt{4k^2+3k}-2k\right\}$. There is no need to switch to the variable x in this case since algebra should be sufficient to find the limit.

2. Find the limit of the sequence $\left\{n\sin(\pi/n)\right\}$. If you choose to use L'Hôpital's Rule to find the limit, be certain that you switch to the variable x.

3. Turn in a solution to problem 5c in Section 3.2. Be careful with the algebra here; I recommend that you write out the first three terms of the sequence. We will be doing lots of work with factorials.

4. Turn in a solution to problem 6 in Section 3.2. Begin by carefully writing out the first four terms of the sequence, then ask yourself how many terms are being added to get the nth term of the sequence and then consider which of the added terms is the smallest.

Name: due March 29

Math 126 **Homework Assignment 20** **Spring 2024**

1. Prove that $f_1^2 + f_2^2 + \cdots + f_n^2 = f_n f_{n+1}$ for each positive integer n.

2. Prove that $f_1 f_2 + f_2 f_3 + f_3 f_4 + \cdots + f_{2n-1} f_{2n} = f_{2n}^2$ for each positive integer n. (A comment concerning this problem was given in class; note that the pattern ends with an even subscript.)

Name: due March 27

Math 126 **Homework Assignment 19** **Spring 2024**

1. Prove that $1^3 + 2^3 + 3^3 + \cdots + n^3 = \dfrac{n^2(n+1)^2}{4}$ for each positive integer n. (Imitate the proof given for Theorem 3.1 in the textbook.)

2. Prove that for each positive integer n, the integer $7^n + 2^{2n+1}$ is divisible by 3. (Imitate the proof of Theorem 3.2 in the textbook.)

Name: due March 6

Math 126 **Homework Assignment 18** **Spring 2024**

1. Use trigonometric substitution to evaluate $\displaystyle\int \frac{\sqrt{x^2+16}}{x^4}\,dx$.

2. Use trigonometric substitution to evaluate $\int \frac{\sqrt{x^2 - a^2}}{x}\, dx$, where a is a positive constant.

Name: due March 4

Math 126 **Homework Assignment 17** **Spring 2024**

1. Evaluate $\displaystyle\int \frac{\sqrt{9x^2-4}}{x^2}\,dx$.

2. Evaluate $\displaystyle\int \frac{1}{\sqrt{9+e^{2x}}}\,dx$.

3. Use integration by parts to establish the reduction formula for secant.

Name: due March 1

Math 126 **Homework Assignment 16** **Spring 2024**

1. Evaluate $\displaystyle\int \frac{8x-3}{\sqrt{16-x^2}}\,dx$.

2. Evaluate $\displaystyle\int \frac{4x-7}{x^2+4x+13}\,dx$.

3. Evaluate $\displaystyle\int \frac{x+3}{\sqrt{21+4x-x^2}}\,dx.$

4. Evaluate $\displaystyle\int \frac{2x^3+14x-5}{x^2+4}\,dx.$

Name: due February 28

Math 126 **Homework Assignment 15** **Spring 2024**

1. Find the length of the curve $y = \dfrac{2x^3}{3} + \dfrac{1}{8x}$ on the interval $[1, 2]$.

2. Find the length of the curve $y = \ln|\cos x|$ on the interval $[0, \pi/3]$. You might find integral 60 in Appendix B helpful.

3. Find the length of the entire curve given by the equation $x^{2/3} + y^{2/3} = 1$. Include a sketch of the curve (it is similar to the curve $x^{1/2} + y^{1/2} = 1$ from a previous assignment), noting that it has branches in all four quadrants.

Name: due February 26

Math 126 **Homework Assignment 14** **Spring 2024**

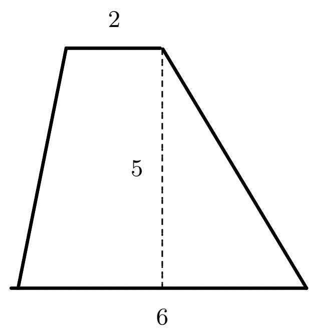
1. Let R be the region that lies under the curve $y = x^3$ and above the x-axis on the interval $[0, a]$, where a is a positive constant. Assuming that this region has constant density, find the center of mass of R.

2. Assuming constant density, find the center of mass of a solid hemisphere of radius r.

Name: due February 23

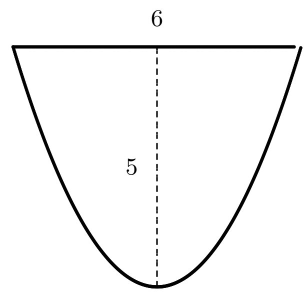
Math 126 **Homework Assignment 13** **Spring 2024**

1. Find the force exerted by a liquid with weight density w on one side of the vertically submerged plate shown below. As indicated, the shape of the plate is a trapezoid. The units on the figure are feet and the top of the plate is four feet beneath the surface of the liquid.



trapezoid

2. Find the force exerted by a liquid with weight density w on one side of the vertically submerged plate shown below. As indicated, the shape of the plate is a parabola. The units on the figure are feet and the top of the plate is four feet beneath the surface of the liquid.



parabola

Name: due February 21

Math 126 **Homework Assignment 12** **Spring 2024**

1. Let R be the region under the graph of $y = \cos x$ and above the x-axis on the interval $[0, \pi/2]$. Find the volume of the solid that is generated when R is revolved around the y-axis.

2. Let R be the region under the graph of $y = 2x^2$ and above the x-axis on the interval $[0, 3]$. Set up, but do not evaluate, an integral that represents the volume of the solid that is generated when R is revolved around (a) the line $x = 3$, (b) the line $y = 20$, (c) the line $x = -1$, and (d) the line $y = -4$.

3. A cylindrical hole of radius r is bored through the center of a sphere with radius $R > r$. The remaining solid resembles a bead since it has a flat top and bottom with a hole through the middle. Find the volume of the bead.

Name: due February 16

Math 126 **Homework Assignment 11** **Spring 2024**

1. Let R be the region that lies below the parabola $y = 18 - \frac{1}{2}x^2$ and above the x-axis. Suppose that R is the base of a solid and that each cross section of the solid taken perpendicular to the y-axis is a semicircle. Find the volume of this solid.

2. Let R be the region under the graph of $y = 8/x$ and above the x-axis on the interval $[2, 4]$. Find the volume of the solid that is generated when R is revolved around the x-axis.

3. Let R be the region bounded by the curves $y = \frac{1}{2}x^2$ and $y = 5x$. Find the volume of the solid that is generated when R is revolved around (a) the x-axis, (b) the y-axis.

Name: due February 14

Math 126 **Homework Assignment 10** **Spring 2024**

1. Find the area of the region bounded by the curves $y = x^4$ and $y = a^3x$, where a is a positive constant.

2. Find the area of the region bounded by the curves $y = e^{2x}$, $y = e^{x/2}$, and $y = 4$. Sketch a careful graph and ponder ways to set up the integral before proceeding to evaluate the integral.

3. Let a be a positive constant. Find the area of the region bounded by the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ and the coordinate axes. If necessary, use an electronic device to obtain a graph of the curve; include a sketch of the region as part of your solution.

Name: due February 7

Math 126 **Homework Assignment 9** **Spring 2024**

1. Evaluate $\displaystyle\int x \sec^2 x \, dx$.

2. Evaluate $\displaystyle\int_0^{\pi} 4x \sin x \, dx$.

3. Find the area of the region that lies under the graph of $y = 2\ln x$ and above the x-axis on the interval $[1, e^3]$. [This problem should end with a complete sentence "The area ..."]

4. Evaluate $\displaystyle\int_2^{\infty} \frac{x^2 + 8x}{x^4}\, dx$. [Be sure to use correct notation for improper integrals.]

Name: due February 5

Math 126 **Homework Assignment 8** **Spring 2024**

1. Evaluate $\displaystyle\int \frac{12\,t}{(2t^2+7)^3}\,dt$.

2. Evaluate $\displaystyle\int \frac{2x+3}{\sqrt{x+2}}\,dx$.

3. Evaluate $\int_0^{\sqrt{2}} t\sqrt{4-t^4}\,dt$. [Remember that the FTC is not always the best way to evaluate an integral.]

4. Evaluate $\displaystyle\int_0^1 \frac{2}{x^{1/2}+x^{3/2}}\,dx$.

Name: due February 2

Math 126 **Homework Assignment 7** **Spring 2024**

1. Evaluate $\int \frac{24}{(4x+1)^2}\,dx$.

2. Evaluate $\int 9\sqrt[3]{2x+7}\,dx$.

3. Evaluate $\displaystyle\int_0^1 \frac{2x+2}{2x^2+4x+1}\,dx$.

4. Evaluate $\displaystyle\int_0^3 (6x-8)\sqrt{9-x^2}\,dx$. (Carefully use the distributive property to split the integral into two integrals, then think carefully about the best way to evaluate each of the integrals.)

Name: due January 31

Math 126 **Homework Assignment 6** **Spring 2024**

1. Evaluate $\displaystyle\int_1^3 \frac{18}{x^2}\, dx$.

2. Evaluate $\displaystyle\int_0^1 \left(2x^3 - \sqrt[4]{x}\right) dx$.

3. Evaluate $\displaystyle\int_0^3 \frac{4}{2x+3}\, dx$.

4. Evaluate $\int_{-2}^{2} 6\sqrt{4-x^2}\,dx$. (Please think first.)

5. Find the area of the region under the curve $y = 6/(1+x^2)$ and above the x-axis on the interval $[-1, 1]$.

6. Evaluate $\int_{0}^{1} 4e^{2x}\,dx$.

Name: due January 29

Math 126 **Homework Assignment 5** **Spring 2024**

1. Find the derivative of the function f defined by $f(x) = \int_0^{3x^2} \sqrt[3]{2t^2 + t}\, dt$.

2. Determine $F''(5)$ given that $F(x) = \int_x^{12} f(t)\, dt$ and $f(x) = \int_1^{4x} \frac{\ln(1+t)}{t}\, dt$.

3. Evaluate $\lim_{x \to 0} \frac{1}{x^5} \int_0^x \left(1 - \cos(t^2)\right) dt$.

4. Find an integral expression for a function f such that $f(2) = 0$ and $f'(x) = 4e^{-x^3}$.

Name: due January 26

Math 126 **Homework Assignment 4** **Spring 2024**

1. Evaluate $\displaystyle\int_0^3 \left(3|x-2| + 8\sqrt{9-x^2}\right)dx$.

2. Referring to Exercise 4 in Section 2.5, find the value of $\int_1^2 \big(4f(x) + 3g(x)\big)\,dx$.

3. Without evaluating either of the integrals, determine which integral is larger and explain why.

$$\int_4^7 \sqrt[4]{x^8 - x - 2}\,dx, \quad \int_4^7 x^2\,dx.$$

Name: due January 24

Math 126 **Homework Assignment 3** **Spring 2024**

1. Use simple facts from geometry to find the area under the graph of each function and above the x-axis on the given interval. Include a sketch of the region whose area is being computed.

   a) $f(x) = 8 - |2x - 4|$ on $[0, 6]$

   b) $g(x) = \sqrt{12x - x^2}$ on $[0, 12]$

2. Use the definition of the integral to express the given integral as a limit of a sum.

a) $\displaystyle\int_2^5 (x^3 - 3x)\,dx$

b) $\displaystyle\int_0^\pi \sin x\,dx$

3. Use the definition of the integral to express the given limit as an integral. For part (b), you need to do some factoring as a first step.

a) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \sqrt{7 + \frac{4i}{n}} \cdot \frac{1}{n}$

b) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{n}{n^2 + i^2}$

Name: due January 22

Math 126 **Homework Assignment 2** **Spring 2024**

1. Express the sum $\dfrac{2}{3} + \dfrac{4}{5} + \dfrac{6}{7} + \cdots + \dfrac{40}{41}$ using summation notation.

2. Find the sum $\displaystyle\sum_{k=11}^{20} k^3$. Do your computations without a calculator.

3. Evaluate $\displaystyle\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{1}{k}-\frac{1}{k+2}\right)$. First find the sum as a function of n using telescoping sums.

4. Use the result of Exercise 2.1.7 to find $\displaystyle\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{1024}$. Simplify your answer.

Name: due January 19

Math 126 **Homework Assignment 1** **Spring 2024**

1. Find an equation for the line tangent to the graph of $y = 5x + \frac{8}{x^2}$ when $x = 2$.

2. Evaluate $\lim_{x \to \infty} \frac{6x^2 + 2x + 9}{\sqrt{2x^4 + 3x^2 + 7}}$.

3. Evaluate $\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$.

4. Find and carefully simplify the derivative of the function f defined by $f(x) = \ln\bigl(x + \sqrt{x^2 + 6}\,\bigr)$.

Name: due January 19

Math 126 **Homework Assignment 0** **Spring 2024**

1. Write down a few things about yourself: for example, hobbies, pets, memorable trips, academic interests

2. Describe one aspect of your experience with mathematics that causes you concern for Calculus II.

3. Describe one aspect of mathematics that you are interested or excited to learn about in Calculus II.