

## Integration Practice Problems

At this stage, we have a limited number of options for finding antiderivatives. The most basic methods we have, along with a simple example of each type, are listed below.

$$\begin{array}{ll} \text{basic formulas} & \int \frac{12}{x^4} dx = -\frac{4}{x^3} + C; \\ \text{guess and check} & \int \frac{12x^2}{x^3 + 15} dx = 4 \ln|x^3 + 15| + C; \\ \text{algebra} & \int \frac{2x + 7}{\sqrt{x}} dx = \int (2x^{1/2} + 7x^{-1/2}) dx = \frac{4}{3}x^{3/2} + 14x^{1/2} + C. \end{array}$$

We also have integration by substitution and integration by parts; examples of these can be found in the textbook and extra notes. It is important to realize that more than one method can be used to find an antiderivative and that it is sometimes necessary to use two or more methods as part of the same problem.

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|--|---|--|
| 1. $\int \frac{1}{x+1} dx$                                     | 2. $\int \frac{x}{x+1} dx$  | 3. $\int \frac{x^2}{x+1} dx$                               |
| 4. $\int \frac{1}{x^2+1} dx$                                   | 5. $\int \frac{x}{x^2+1} dx$  | 6. $\int \frac{x^2}{x^2+1} dx$                             |
| 7. $\int \frac{1}{4x^2+1} dx$                                  | 8. $\int \frac{1}{x^2+4} dx$  | 9. $\int \frac{x}{(x+4)^2} dx$                             |
| 10. $\int \frac{x^2}{\sqrt[3]{x^3-4}} dx$                      | 11. $\int \frac{x}{\sqrt{4-x^2}} dx$                                  | 12. $\int \frac{1}{\sqrt{4-x^2}} dx$                       |
| 13. $\int \cos^4(3x) \sin(3x) dx$                              | 14. $\int \tan^3 x \sec^2 x dx$                                       | 15. $\int \cot x dx$                                       |
| 16. $\int x \sin(3x) dx$                                       | 17. $\int x \sin(2x^2) dx$  | 18. $\int \frac{\sin(\ln x)}{x} dx$                        |
| 19. $\int \frac{x^2 + 3x - 1}{x} dx$                           | 20. $\int \frac{3x - 4}{\sqrt{x}} dx$                                 | 21. $\int (x^3 + 1)^2 dx$                                  |
| 22. $\int \frac{x + 2}{3x^2 + 12x - 8} dx$                     | 23. $\int x \ln x dx$   | 24. $\int \frac{\ln x}{x} dx$                              |
| 25. $\int x \sin(x^2) \cos(x^2) dx$                            | 26. $\int x e^{-x/5} dx$  | 27. $\int \frac{1}{1 + \sqrt{x}} dx$                       |
| 28. $\int_0^9 \sqrt{81 - x^2} dx$                              | 29. $\int_0^9 x \sqrt{81 - x^2} dx$                                   | 30. $\int_0^3 x \sqrt{81 - x^4} dx$                        |
| 31. $\int_1^2 \frac{4}{x^2} dx$                                | 32. $\int_0^3 \frac{x}{\sqrt{x^2 + 16}} dx$                           | 33. $\int_0^5 3x \sqrt{x + 4} dx$                          |
| 34. $\int_0^5 \frac{6x + 7}{\sqrt{3x + 1}} dx$ (use $u=3x+1$ ) | 35. $\int_0^5 \frac{6x + 7}{\sqrt{3x + 1}} dx$ (use $u=\sqrt{3x+1}$ ) | 36. $\int_0^5 \frac{6x + 7}{\sqrt{3x + 1}} dx$ (use parts) |
| 37. $\int_0^5  x^3 - 16x  dx$                                  | 38. $\int_0^4  x^2 - 3x + 2  dx$                                      | 39. $\int_0^4  2 - e^{x/2}  dx$                            |

The answers to the definite integrals are  $81\pi/4$ , 243,  $81\pi/8$ , 2, 1, 101.2, 38, 38, 38, 84.25,  $17/3$ , and  $2e^2 + 8 \ln 2 - 14$ . You should check the indefinite integrals by differentiating your proposed antiderivative.