Review for exam on integration

Let R be the region bounded by the curves $y = 3^x$ and $y = 4x + 1$. For each of the following, write an expression involving one or more integrals that represents the requested quantity. (There is no need to evaluate the integrals.)

1. the volume generated when R is revolved around the x-axis
2. the volume generated when R is revolved around the y-axis
3. the volume generated when R is revolved around the line $x = 2$
4. the volume generated when R is revolved around the line $x = -4$
5. the volume generated when R is revolved around the line $y = 9$
6. the volume generated when R is revolved around the line $y = -2$
7. the volume of the solid whose base is R and each cross-section of the solid taken perpendicular to the x-axis is a square
8. the volume of the solid whose base is R and each cross-section of the solid taken perpendicular to the y-axis is a semicircle
9. the force of a liquid on the vertically submerged plate R assuming that the surface of the liquid is $y = 12$
10. the perimeter of R
11. the x-coordinate of the center of mass of R
12. the y-coordinate of the center of mass of R

Evaluate each of the following integrals. (Note that some of these are difficult.)

13. $\displaystyle\int \frac{4x+7}{x^2+8x+17}\,dx$
14. $\displaystyle\int \frac{x+1}{\sqrt{6x-x^2}}\,dx$
15. $\displaystyle\int \frac{2x+5}{x+6}\,dx$
16. $\displaystyle\int \frac{x}{e^x}\,dx$
17. $\displaystyle\int \frac{x}{e^{2x^2}}\,dx$
18. $\displaystyle\int x\sin(x/2)\,dx$
19. $\displaystyle\int \arcsin x\,dx$
20. $\displaystyle\int x\arcsin x\,dx$
21. $\displaystyle\int (\arcsin x)^2\,dx$
22. $\displaystyle\int \tan^3 x\,dx$
23. $\displaystyle\int \frac{4x-3}{2x^2+5x-7}\,dx$
24. $\displaystyle\int \frac{x^2+2x+4}{x^3+x^2+x+1}\,dx$
25. $\displaystyle\int \tan^4 x\sec^4 x\,dx$
26. $\displaystyle\int \frac{\sqrt{x^2-1}}{x^4}\,dx$
27. $\displaystyle\int \frac{\sqrt{25-(\ln x)^2}}{x}\,dx$
28. Convert $\displaystyle\int \frac{(x^2+4)^{3/2}}{x^5}\,dx$ to a trigonometric integral but do not continue any further.
29. Suppose that $x = 2\tan\theta$. Represent $\sin\theta + 4\sec\theta + 3\theta$ in terms of x.
30. Find the length of the curve $y = \frac{4}{5}x^{5/4}$ on the interval $[0, 9]$.
31. Use Simpson's rule with $n = 4$ to approximate $\displaystyle\int_1^5 \frac{12}{x^2}\,dx$.

1. $V = \pi \displaystyle\int_0^2 \big((4x+1)^2 - (3^x)^2\big)\,dx$

2. $V = 2\pi \displaystyle\int_0^2 x(4x+1-3^x)\,dx$

3. $V = 2\pi \displaystyle\int_0^2 (2-x)(4x+1-3^x)\,dx$

4. $V = 2\pi \displaystyle\int_0^2 (x+4)(4x+1-3^x)\,dx$

5. $V = \pi \displaystyle\int_0^2 \big((9-3^x)^2 - (8-4x)^2\big)\,dx$

6. $V = \pi \displaystyle\int_0^2 \big((4x+3)^2 - (2+3^x)^2\big)\,dx$

7. $V = \displaystyle\int_0^2 (4x+1-3^x)^2\,dx$

8. $V = \dfrac{\pi}{8} \displaystyle\int_1^9 \Big(\frac{\ln y}{\ln 3} - \frac{y-1}{4}\Big)^2\,dy$

9. $F = \displaystyle\int_1^9 w(12-y)\Big(\frac{\ln y}{\ln 3} - \frac{y-1}{4}\Big)\,dy$

10. $P = \sqrt{68} + \displaystyle\int_0^2 \sqrt{1+(3^x \ln 3)^2}\,dx$

11. $\overline{x} = \displaystyle\int_0^2 x\rho(4x+1-3^x)\,dx \div \int_0^2 \rho(4x+1-3^x)\,dx$

12. $\overline{y} = \displaystyle\int_1^9 y\rho\Big(\frac{\ln y}{\ln 3} - \frac{y-1}{4}\Big)\,dy \div \int_1^9 \rho\Big(\frac{\ln y}{\ln 3} - \frac{y-1}{4}\Big)\,dy$

13. $2\ln(x^2+8x+17) - 9\arctan(x+4) + C$

14. $-\sqrt{6x-x^2} + 4\arcsin((x-3)/3) + C$

15. $2x - 7\ln|x+6| + C$

16. $-(x+1)e^{-x} + C$

17. $-\dfrac{1}{4}\,e^{-2x^2} + C$

18. $-2x\cos(x/2) + 4\sin(x/2) + C$

19. $x\arcsin x + \sqrt{1-x^2} + C$

20. $\dfrac{1}{4}\big(2x^2\arcsin x + x\sqrt{1-x^2} - \arcsin x\big) + C$

21. $x(\arcsin x)^2 + 2\sqrt{1-x^2}\arcsin x - 2x + C$

22. $\dfrac{1}{2}\tan^2 x + \ln|\cos x| + C$

23. $\dfrac{1}{9}\ln|x-1| + \dfrac{17}{9}\ln|2x+7| + C$

24. $\dfrac{3}{2}\ln|x+1| - \dfrac{1}{4}\ln(x^2+1) + \dfrac{5}{2}\arctan x + C$

25. $\dfrac{1}{5}\tan^5 x + \dfrac{1}{3}\tan^3 x + C$

26. $\dfrac{(x^2-1)^{3/2}}{3x^3} + C$

27. $\dfrac{1}{2}\ln x\sqrt{25-(\ln(x))^2} + \dfrac{25}{2}\arcsin((\ln x)/5) + C$

28. $\dfrac{1}{2}\displaystyle\int \csc^5\theta\,d\theta$

29. $\dfrac{x}{\sqrt{x^2+4}} + 2\sqrt{x^2+4} + 3\arctan(x/2)$

30. $L = \dfrac{232}{15}$

31. $S_4 = 10 + \dfrac{11}{225} \approx 10.048889$ compared to $\displaystyle\int_1^5 \frac{12}{x^2}\,dx = 9.6$