Total: Name:

Math 225 **Introductory Assignment** **Spring 2014**

Write clear and accurate solutions to each of the following problems in the style expected for this class (see the syllabus). You should not use any electronic devices as an aid in solving these problems.

1. Find an equation for the line tangent to the graph of $f(x) = \dfrac{4x}{(3x+1)^2}$ when $x = 1$.

note that $f'(x) = \dfrac{(3x+1)^2 \cdot 4 - 4x \cdot 2(3x+1)\cdot 3}{(3x+1)^4} = \dfrac{4(3x+1-6x)}{(3x+1)^3} = \dfrac{4(1-3x)}{(3x+1)^3}$.

Since $f(1) = \frac{1}{4}$ and $f'(1) = -\frac{1}{8}$, the tangent line goes through the point $(1, \frac{1}{4})$ and has a slope of $-\frac{1}{8}$. An equation for the tangent line is thus $y - \frac{1}{4} = -\frac{1}{8}(x-1)$ or $x + 8y = 3$.

2. Find the point guaranteed by the Mean Value Theorem for the function $f(x) = x^3 - 4x^2 + 7$ on the interval $[0, 2]$.

Since $f'(x) = 3x^2 - 8x$ and $\dfrac{f(2)-f(0)}{2-0} = \dfrac{-1-7}{2} = -4$, we want to find $c \in (0,2)$ so that $3c^2 - 8c = -4$. Solving yields $3c^2 - 8c + 4 = (3c-2)(c-2) = 0$ or $c = \frac{2}{3}, 2$. The point guaranteed by the Mean Value Theorem is $\frac{2}{3}$.

3. Find the minimum value of the function $f(x) = x + \dfrac{8}{x^2}$ on the interval $[1, 5]$.

The minimum value occurs at a critical point or the endpoints.

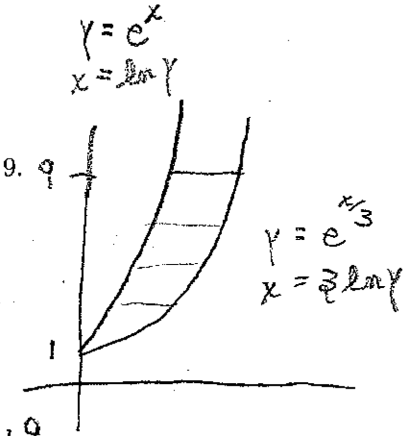
$f'(x) = 1 - \dfrac{16}{x^3}$ $\quad f'(x) = 0 \Rightarrow x = \sqrt[3]{16} = 2\sqrt[3]{2}$.

The only critical point is $2\sqrt[3]{2}$. Referring to the table

x	$f(x)$
1	9
$2\sqrt[3]{2}$	$2\sqrt[3]{2} + \dfrac{8}{4\sqrt[3]{4}} = 3\sqrt[3]{2}$
5	$5\frac{8}{25} = 5.32$

, we see that the minimum value of f on $[1,5]$ is $3\sqrt[3]{2}$.

4. Find the area of the region bounded by the curves $y = e^x$, $y = e^{x/3}$, and $y = 9$.



The region is sketched in the figure and is most easily described horizontally. The area is given by

$$\int_1^9 (3\ln y - \ln y)\,dy = 2\int_1^9 \ln y\,dy = 2(y\ln y - y)\Big|_1^9 \qquad \text{(use parts)}$$

$$= 2\big((9\ln 9 - 9) - (0 - 1)\big) = 2(9\ln 9 - 8)$$

The area of the region is $36\ln 3 - 16$ square units.

5. Evaluate $\displaystyle\int \frac{x}{x^2 + 4x + 13}\,dx$.

$$\int \frac{x}{x^2+4x+13}\,dx = \int \frac{x}{(x+2)^2+9}\,dx$$

complete the square

let $u = x+2$

$$= \int \frac{u-2}{u^2+9}\,du$$

split up

$$= \int \left(\frac{u}{u^2+9} - \frac{2}{u^2+9}\right) du$$

formulas

$$= \frac{1}{2}\ln|u^2+9| - \frac{2}{3}\arctan\frac{u}{3} + C$$

$$= \frac{1}{2}\ln|x^2+4x+13| - \frac{2}{3}\arctan\frac{x+2}{3} + C$$

6. Evaluate $\displaystyle\int_0^{\pi/3} \cos^3 x\,dx$.

$$\int_0^{\pi/3} \cos^3 x\,dx = \int_0^{\pi/3} \cos^2 x \cdot \cos x\,dx$$

$$= \int_0^{\pi/3} (1 - \sin^2 x)\cos x\,dx$$

$$= \int_0^{\pi/3} (\cos x - \sin^2 x\cos x)\,dx$$

$$= \left(\sin x - \frac{1}{3}\sin^3 x\right)\Big|_0^{\pi/3}$$

$$= \frac{\sqrt{3}}{2} - \frac{1}{3}\cdot\frac{3\sqrt{3}}{8}$$

$$= \frac{3\sqrt{3}}{8}$$