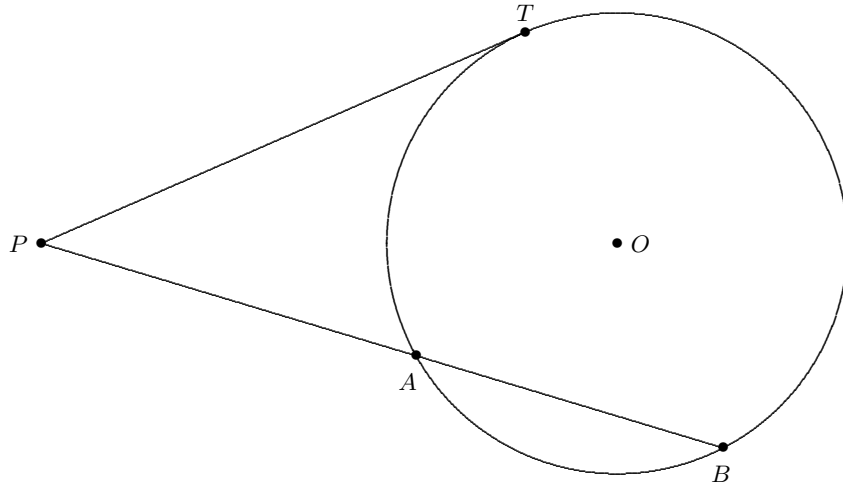
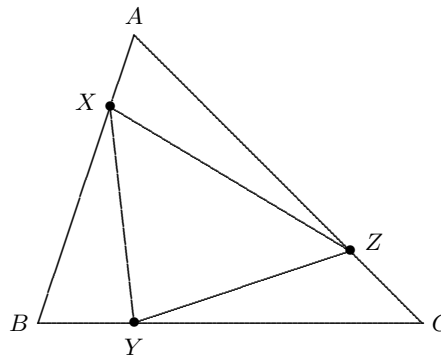


Sample (Novel) Midterm Problems

- In the figure below, line PT is a tangent line for the circle centered at O and line PAB is a secant line for the circle. We proved in class that $PT \cdot PT = PA \cdot PB$ using similar triangles. It is possible to give a proof that only uses the Pythagorean Theorem and some basic properties of circles. Provide such a proof. As a suggestion, draw four lines out of the center O , one of which bisects the segment AB , and start looking for right triangles.

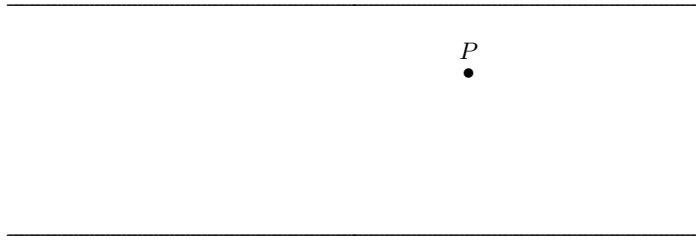


- Points X , Y , and Z are located on sides AB , BC , and CA , respectively, of $\triangle ABC$ in such a way that the three ratios AX/AB , BY/BC , and CZ/CA are all equal. Call this common value r , where $0 < r < 1/2$. Find K_{XYZ}/K_{ABC} (as a function of r), the ratio of the areas of the two triangles.

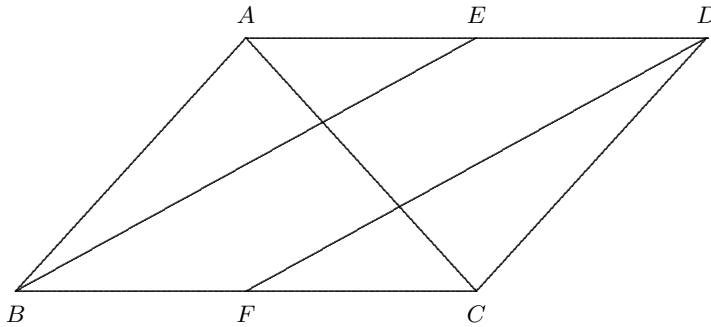


- Prove: If from a variable point on the base of an isosceles triangle parallels to the sides are drawn, a parallelogram is formed whose perimeter is independent of the choice of the point. (Part of the problem is to interpret these words and draw an appropriate picture.)
- Show how to construct an equilateral triangle whose area is equal to that of a given square.

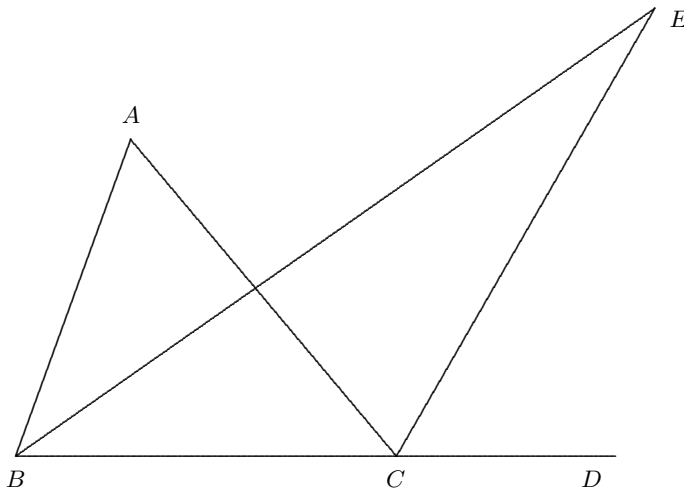
5. Given two parallel lines and an arbitrary point between them, show how to construct a circle through the given point and tangent to the two given lines. (One of several possibilities is illustrated below.)



6. In the figure below, points E and F are the midpoints of sides AD and BC , respectively, of parallelogram $ABCD$. Prove that lines BE and DF trisect diagonal AC .



7. The line BE bisects angle B of triangle ABC and the line CE bisects the exterior angle ACD . Prove that angle E is half of angle A .



8. In the figure, we are given quadrilateral $ABCD$ with diagonal AC not passing through the midpoint M of diagonal BD . Let G_1 and G_2 be the centroids of triangles ABD and BCD , respectively. These points are indicated by bullets in the figure. Prove that the line G_1G_2 is parallel to the line AC .

