I certify that I did not use any computational device for symbolic manipulation or data storage.

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Signature

Total: Name:

Math 244 **First Exam** **Spring 2010**

Write neat, concise, and accurate solutions to each of the following problems in the space provided. Include all of the relevant details and intermediate steps, with brief explanations as necessary, and conclude each problem with a complete sentence. Check your computations carefully. Each problem is worth 10 points.

1. Provide a brief answer for each of the following.
   a) Give an example of a second order nonlinear ordinary differential equation. (A simple one is fine, but avoid a trivial example.)
   b) Write the standard form for a first order linear ordinary differential equation.
   c) Determine the largest open interval in which a solution to the following initial value problem has a solution.
   $$(\sin t)y' - 4y = e^{2t}, \quad y(1) = 4$$

2. Solve the initial value problem: $2y' + y = 6t + 5, \quad y(0) = 12$.

3. Solve the initial value problem $t^2y' = 4y^3, \quad y(1) = -1$ explicitly for y, then determine the interval on which the solution is valid.

4. Find the function y that satisfies the initial value problem $y' = 0.01\,y\,(600 - y), \quad y(0) = 100$, then estimate $y(0.5)$ to the nearest hundredth.

5. Recall that Newton's Law of Cooling states that the rate of change of the temperature of an object is proportional to the difference between the temperature of the object and the temperature of its surroundings. An object with a temperature of 85° C is brought into a large room where the temperature is 20° C. Five minutes later, the temperature of the object is 70° C. Assuming that Newton's Law of Cooling applies to this object, how much more time is required for the temperature of the object to reach 30° C? Record your answer to the nearest tenth of a minute.

6. A 1200 gallon tank contains 600 gallons of fresh water. A brine containing one-fourth pound of salt per gallon of water runs into the tank at the rate of six gallons per minute, and the well-stirred mixture runs out of the tank at the rate of three gallons per minute. Determine the concentration of salt in the tank at the instant the tank overflows.

1c. $(0, \pi)$
2. $y = 6t - 7 + 19e^{-t/2}$
3. $y = -\sqrt{t/(8 - 7t)}$ so the interval is $[0, 8/7)$.
4. $y = 600e^{6t}/(5 + e^{6t})$
5. about 30.7 additional minutes
6. 3/16 pounds per gallon

I certify that I did not use any electronic device for symbolic manipulation or data storage.

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Signature

Total: Name:

Math 244 **First Exam** **Spring 2011**

Write neat, concise, and accurate solutions to each of the following problems in the space provided. Include all of the relevant details and intermediate steps, with brief explanations as necessary, and conclude each problem with a complete sentence. Check your computations carefully. Each problem is worth 12 points.

1. Solve the initial value problem $8y' + y = 15e^{-2t}$, $y(0) = 1$, then determine the maximum value of the solution for $t \geq 0$. Give your answer to the nearest thousandth.
2. A 500 gallon tank contains 400 gallons of fresh water. A brine containing one pound of salt per gallon of water runs into the tank at the rate of r gallons per minute, and the well-stirred mixture runs out of the tank at the same rate. Find a value for r so that the concentration of salt in the tank after 40 minutes is one-half pound per gallon.
3. Solve the initial value problem $y' = t(1 - y^2)$, $y(0) = 0$ then answer the following two questions. Give exact answers rather than decimal approximations.
   a) Find the value of $t > 0$ for which $y = 0.95$.
   b) Find the value of y when $t = 1$.
4. Suppose that a certain population grows at a rate proportional to the square root of the current size of the population. Assume that the population is initially 400 and that one year later the population is 625. Determine the time in which the population reaches 10000.

I certify that the solutions written below (take-home part) are my own work. _______________

Signature

Write a neat, concise, and accurate solution for the problem on this page; partial credit will be kept to a minimum. Include all relevant details, make certain all of the steps are clear, use correct notation, and use complete sentences when appropriate. I strongly recommend that you work the problem first on a separate page then copy a more polished solution onto this page (use the space wisely). You are expected to work independently on this test (remember that blue plagiarism form you signed as well as the statement above) but you may use the textbook and your notes. This problem is due by noon on Friday (2/11); either slide it under my office door or put it in my Olin Hall mailbox.

5. This problem refers to Exercise 19 in Section 2.5; you do not need to copy the problem or do parts (a) and (b) here. (For the record, the problem is the same in both the 8th and 9th editions.) Assume that the cylindrical tank has a radius of four feet and a height of five feet and that the tank is initially empty. Use the values $k = 4\pi$ (with units cubic feet per minute), $a = \pi/144$ (with units square feet), and $\alpha = 0.6$, and note carefully that g then has a value of $32 \cdot 3600$ feet per minute^2. Solve the differential equation to find an equation involving the height h (in feet) and the time t (in minutes). For the record, an integral of the form $\int \frac{1}{b - \sqrt{x}}\, dx$ is easily determined with the substitution $u = b - \sqrt{x}$ or $x = (b - u)^2$. Then answer the following two questions.
   a) Find the depth of water in the tank after 30 minutes.
   b) Determine the time required for the tank to reach 90% of its equilibrium depth.

1. The maximum value of y is $\frac{15}{16}\sqrt[5]{16}$.
2. The required rate is $10 \ln 2$ gallons per minute.
3. We find that $t = \sqrt{\ln 39}$ and $y = (e - 1)/(e + 1)$.
4. The population reaches 10000 after 16 years.
5. The depth is about 2.75 feet and the time is about 64.7 minutes.