**Math 244: Review for Second Exam**

The second exam will cover Chapter 3 (with perhaps a little bit of Chapter 7 tossed in). Be certain to know the concepts and techniques that we have discussed in this chapter. In addition, you should know (as in be able to write out on an exam) a proof of Theorem 3.2.2 (principle of superposition), a proof of Theorem 3.2.6 (Abel's Theorem), and a derivation for the formulas that arise for variation of parameters. You should also be familiar with the terms that appear in the sections with applications. Remember that it is important that you have done **all** of the assigned problems as listed on the assignments page for the class.

Here are some further problems you can work on; you should be able to solve these fairly quickly.

1. Find the general solution to each of the following.
   a. $y'' + 2y' - 8y = 0$
   b. $y'' + 2y' + y = 0$
   c. $y'' + 2y' + 4y = 0$
2. Find one solution to each of the following. (Check your answers.)
   a. $y'' + 2y' + 3y = t^2 - 4t$
   b. $y'' + 2y' + 3y = 2\sin t$
   c. $y'' + 2y' + 3y = e^{2t}$
3. Use your answers to the previous problem to find the general solution to $y'' + 2y' + 3y = 4\sin t - 3e^{2t}$.
4. Suppose that $y_1$ and $y_2$ are solutions to the initial value problems

   $$2y'' - ty' + 4y = 0,\ y(0) = 2,\ y'(0) = 1 \quad \text{and} \quad 2y'' - ty' + 4y = 0,\ y(0) = 1,\ y'(0) = 5,$$

   respectively. Find $W(y_1, y_2)(t)$. (There should be no constant in your answer.)
5. Solve the initial value problem $y'' + 4y = t,\ y(0) = 0,\ y'(0) = 1$.
6. Let $f$ and $g$ be two differentiable functions defined on the interval $[0, 6]$. Suppose that $W(f, g)(1) = 4$. Find the value of $W(3f, f + 2g)(1)$.

4. $W(y_1, y_2) = 9e^{t^2/4}$
5. $y = \dfrac{3}{8}\sin(2t) + \dfrac{1}{4}t$
6. $W(3f, f + 2g)(1) = 24$

Total: Name:

**Math 244** **Second Exam** **Spring 2010**

Write neat, concise, and accurate solutions to each of the following problems in the space provided. Include all of the relevant details and intermediate steps, with brief explanations as necessary, and conclude each problem with a complete sentence. Check your computations carefully. No calculators are allowed for this exam. Each problem is worth 10 points.

1. Solve the initial value problem $y'' + 4y' + 10y = 0, \quad y(0) = 1, \quad y'(0) = 2.$
2. Find the general solution of the differential equation $y'' - 2y' + y = 5\sin(2t).$
3. The general solution of a second order linear differential equation with constant coefficients is given by $y = ce^t + de^{-3t} - 2t$, where $c$ and $d$ are arbitrary constants. Find the differential equation.
4. Consider the second order linear differential equation $y'' + p(t)y' + q(t)y = g(t)$. Suppose that $w$ is a solution to this equation and that $\phi$ is a solution to the corresponding homogeneous differential equation. Prove that $y = c\,\phi + w$ is a solution to the original differential equation for any constant value $c$. Be certain to include all of the steps of the proof. (This is not the proof I asked you to learn, but it uses the same ideas as the principle of superposition and it is these ideas that are the most important.)
5. A mass weighing 6 lb stretches a spring 4 in. Suppose that the mass is attached to a viscous damper with a damping coefficient of 3 lb-sec/ft. The mass is pulled down 2 in. from its equilibrium position and given a downward velocity of 3 in/sec.
   a) Set up, but do not solve, an initial value problem for the position $u$ of the mass at any time $t \geq 0$.
   b) Determine the quasi-period of this damped motion. (Note that there is no need to find the full solution to the IVP.)
6. Find the general solution to $t^2y'' + 5ty' + 4y = 0$, valid for $t > 0$. (One solution is of the form $t^r$ for some value of $r$; substituting this expression into the differential equation allows you to find $r$. Then use Abel's Theorem to find a second solution.)

1. The solution to the IVP is $y = e^{-2t}\Big(\cos\big(\sqrt{6}\,t\big) + \dfrac{4}{\sqrt{6}}\sin\big(\sqrt{6}\,t\big)\Big)$.
2. The general solution is $y = ce^t + dte^t + \frac{4}{5}\cos 2t - \frac{3}{5}\sin 2t$, where $c$ and $d$ are constants.
3. A differential equation with the desired general solution is $y'' + 2y' - 3y = 6t - 4$.
4. Be certain you are clear on the steps required for proofs like this.
5. The initial value problem is (be careful with units) $\frac{3}{16}u'' + 3u' + 18u = 0,\ u(0) = \frac{1}{6}, u'(0) = \frac{1}{4}$. The quasi-period of the damped motion is $\sqrt{2}\,\pi/4$.
6. The general solution for $t > 0$ is $y = ct^{-2} + dt^{-2}\ln t$.

Total: Name:

**Math 244** **Second Exam** **Spring 2011**

Write neat, concise, and accurate solutions to each of the following problems in the space provided. Include all of the relevant details and intermediate steps, with brief explanations as necessary, and conclude each problem with a complete sentence. Check your computations carefully. No calculators are allowed for this exam. Each problem is worth 10 points.

1. Find the general solution of the differential equation $2y'' - y' - 10y = 0$.

2. Find the maximum value of the function that satisfies the initial value problem

$$y'' + 2y' + y = 0, \quad y(0) = 0, \quad y'(0) = 4.$$

   Be certain to verify (there are several ways) that you do indeed have a maximum value.

3. Suppose that $v$ is a solution to the differential equation $y'' + p(t)y' + q(t)y = g(t)$ and that $w$ is a solution to the differential equation $y'' + p(t)y' + q(t)y = h(t)$ (the only difference is the nonhomogeneous part). Prove (using a variation of the proof for the principle of superposition) that $y = v - 2w$ is a solution to the differential equation $y'' + p(t)y' + q(t)y = g(t) - 2h(t)$. Be certain to include all of the steps (probably four of them) of the proof.

4. Solve the initial value problem $y'' + 4y = 4e^{2t}, \quad y(0) = 1, \quad y'(0) = 3.$

5. Consider the initial value problem

$$(1 - x^2)y'' + 2xy' - 2y = 0, \quad y(0) = 4, \quad y'(0) = 6.$$

   Find the solution $y$ to this IVP then find $y(0.5)$. Note that $x$ is one solution to the differential equation.

6. The position $u$ of a certain spring-mass system (in mks units) is determined by the initial value problem

$$u'' + \tfrac{1}{2}u' + 2u = 5\cos t, \quad u(0) = 0, \quad u'(0) = 1.$$

   Answer the following questions about this motion, noting that you only need to do enough work to answer the questions. However, I will not accept answers that are simply written down using memorized formulas involving the parameters $m$, $\gamma$, and $k$.

   a) Determine the quasi-period of the transient solution.

   b) Determine the amplitude of the steady-state solution.

1. The general solution is $y = ce^{-2t} + de^{5t/2}$, where $c$ and $d$ are constants.
2. The maximum value of the solution is $4/e$.
3. Be certain you are clear on the steps required for proofs like this.
4. The solution to the IVP is $y = \frac{1}{2}\cos(2t) + \sin(2t) + \frac{1}{2}e^{2t}$.
5. The solution to the IVP is $y = 4x^2 + 6x + 4$ and thus $y(\frac{1}{2}) = 8$.
6. The quasi-period of the transient solution is $8\pi/\sqrt{31}$ and the amplitude of the steady-state solution is $\sqrt{20}$. Two comments are worth mentioning. You should not find the solution to this IVP as it takes a while and is not needed to answer the two questions. You also do not need to memorize formulas in terms of the parameters $m$, $\gamma$, and $k$ for the quantities requested in this problem.