Total: Name:

Math 244 **Third Exam** **Spring 2010**

Write neat, concise, and accurate solutions to each of the following problems in the space provided. Include all of the relevant details and intermediate steps, with brief explanations as necessary, including an appropriate conclusion. Check your computations carefully. No calculators are allowed for this exam. Each problem is worth 7 points.

1. Express the power series $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k 4^k}{(2k)!}\, x^{2k+1}$ in terms of familiar functions.
2. Find the radius of convergence for the power series $\displaystyle\sum_{k=0}^{\infty} \frac{(k!)^2}{3^k (2k)!}\, x^k$. (Be careful.)
3. Determine a lower bound for the radius of convergence of a power series solution centered at 1 for the differential equation $(x^2+2x+5)\, y'' + (x^2+3x)\, y' - x\, y = 0$.
4. Solve the initial value problem $3x^2y'' - 2xy' - 2y = 0$, $y(1) = 4$, $y'(1) = 1$. (Note that we have simple methods to solve this type of equation.)
5. Use the derivative method to find the terms up through x^6 (with simplified coefficients) of a power series solution centered at 0 for the initial value problem $y'' + y' - 4xy = 0$, $y(0) = 1$, $y'(0) = 0$.
6. The recurrence relation satisfied by the coefficients a_k of the power series solution centered at 0 for a certain second order linear differential equation is given by
$$a_{k+2} = \frac{-3}{k+2}\, a_k.$$
Assuming that the initial conditions are $y(0) = 1$ and $y'(0) = 0$, find the complete power series solution. You should represent the coefficients in simplest terms.
7. Use the power series method (that is, use infinite series and sigma notation, etc.) to find the recurrence relation satisfied by the coefficients of the power series solution centered at 0 for the differential equation
$$y'' - 2x^2y' + 5y = 0.$$
Represent the recurrence relation as an equation that gives one coefficient in terms of one or more previous coefficients. Do NOT go any farther toward solving the equation.
8. Consider the differential equation $x^2(1-x^2)y'' + 2xy' - 2y = 0$. Show that 0 is a regular singular point for this equation, then find (preferably using information you just computed) the exponents at this singularity.

1. $x\cos(2x)$
2. $\rho = 12$
3. $\sqrt{8}$
4. $y = 3x^{-1/3} + x^2$
5. $y = 1 + \frac{2}{3}x^3 - \frac{1}{6}x^4 + \frac{1}{30}x^5 + \frac{1}{12}x^6 + \cdots$
6. $y = \displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k 3^k}{2^k k!}\, x^{2k}$ and you should be able to identify this function.
7. $a_{k+2} = \dfrac{2(k-1)a_{k-1} - 5a_k}{(k+2)(k+1)}$ for $k \geq 1$.
8. 1 and -2

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Math 244 **Third Exam** **Spring 2011**

Write neat, concise, and accurate solutions to each of the following problems in the space provided. Include all of the relevant details and intermediate steps, with brief explanations as necessary, including an appropriate conclusion. Check your computations carefully. No calculators are allowed for this exam. Each problem is worth 7 points.

1. Find the radius of convergence for the power series $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k(2k)!}{5^k(k!)^2}\, x^k$.
2. Determine a lower bound for the radius of convergence of a power series solution centered at 3 for the differential equation $(x^2+4x+5)y'' - 3xy' + 2y = 0$.
3. By making a clever observation (think product rule), solve the initial value problem
$$y'' + 4xy' + 4y = 0, \quad y(0) = 1, \quad y'(0) = 0.$$
4. Solve the initial value problem $\quad 3x^2y'' + 14xy' - 4y = 0, \quad y(1) = 8, \quad y'(1) = -6.$
5. Use the derivative method to find the terms up through x^7 (with simplified coefficients) of a power series solution centered at 0 for the initial value problem $y'' - xy' + 2y = 0, \quad y(0) = 1, \quad y'(0) = 2.$
6. The recurrence relation satisfied by the derivatives $d_k = y^{(k)}(0)$ of the power series solution centered at 0 for a certain second order linear differential equation is given by $d_k = \dfrac{-4}{k}\, d_{k-2}$ for $k \geq 2$. Assuming that the initial conditions are $y(0) = 0$ and $y'(0) = 1$, find the complete power series solution. Your final answer should be represented in sigma notation with a clear formula for the coefficients.
7. Use the power series method (that is, use infinite series and sigma notation, etc.) to find the recurrence relation satisfied by the coefficients of the power series solution centered at 0 for the differential equation
$$y'' - 3y' + 2xy = 0.$$
Represent the recurrence relation as an equation that gives one coefficient in terms of one or more previous coefficients. Do NOT go any farther toward solving the equation.
8. Consider the differential equation $(x^3 - 2x^2)y'' - 7xy' + 12y = 0$. Show that 0 is a regular singular point for this equation, then find the exponents at this singularity (preferably without recourse to infinite series).

1. $\rho = 5/4$
2. $\sqrt{26}$
3. $y = e^{-2x^2}$
4. $y = 6x^{1/3} + 2x^{-4}$
5. $y = 1 + 2x - x^2 - \frac{1}{3}x^3 - \frac{1}{60}x^5 - \frac{1}{840}x^7 - \cdots$
6. $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k 8^k k!}{((2k+1)!)^2}\, x^{2k+1}$
7. $a_{k+2} = \dfrac{3(k+1)a_{k+1} - 2a_{k-1}}{(k+2)(k+1)}$ for $k \geq 1$.
8. -4 and $3/2$