Total: Name:

Math 244 **Fourth Exam** **Spring 2010**

Write neat, concise, and accurate solutions to each of the following problems in the space provided. Include all of the relevant details and intermediate steps, with brief explanations as necessary, including an appropriate conclusion. As this exam is relatively short (that is the goal), you should pay particular attention to your writing and notation. Check your computations carefully. No calculators are allowed for this exam. Each problem is worth 10 points.

1. Find the Laplace transforms of each of the following three functions; pay careful attention to your notation for these functions.
$$f(t) = 3e^{4t}; \qquad g(t) = e^{-2t}\cos(3t); \qquad h(t) = u_3(t)(t+1).$$

2. Suppose that $Y(s)$ is the Laplace transform of a function $y(t)$. Find $y(t)$ if $Y(s) = \dfrac{2s-1}{s^2+6s+13}$. You need to be certain that all of your steps are written carefully.

3. Suppose that a (sufficiently nice) function f satisfies $f(0) = 1$, $f'(0) = 2$, and $F(3) = 5$, where F is the Laplace transform of f. Let g be the function defined by $g(t) = f''(t) + 2f'(t) + 4f(t)$ and let G be the Laplace transform of g. Find $G(3)$. Your solution should clearly indicate the formula for the function $G(s)$ and the computations you used to find $G(3)$.

4. Consider the function f given by the Maclaurin series $f(t) = \displaystyle\sum_{k=0}^{\infty} \frac{3^k}{(k!)^2}\, t^k$. Find the Laplace transform of f AND represent the transform in terms of familiar calculus functions. (You may assume that the radius of convergence of the Maclaurin series for f is infinity and that term by term operations are valid.) Represent your series correctly and indicate how you arrive at your final answer.

5. Use the method of Laplace transforms to solve the initial value problem
$$y'' - 2y' - 15y = 2e^t, \quad y(0) = 0, \quad y'(0) = 1.$$
Make certain that you define the functions that appear in your work.

6. Consider the initial value problem
$$y'' + y = k\,\delta(t-4\pi), \quad y(0) = 3, \quad y'(0) = 1,$$
where k is a positive constant. Solve this initial value problem in terms of k, then choose a value for k so that the amplitude of the solution for $t > 4\pi$ has a value of 5.

1. $F(s) = \dfrac{3}{s-4},\ s > 4; \qquad G(s) = \dfrac{s+2}{(s+2)^2+9},\ s > -2; \qquad H(s) = \left(\dfrac{1}{s^2} + \dfrac{4}{s}\right)e^{-3s},\ s > 0.$
2. $y(t) = \frac{1}{2}e^{-3t}\big(4\cos(2t) - 7\sin(2t)\big)$.
3. $G(3) = 88$.
4. $\mathcal{L}\{f(t)\}(s) = \dfrac{1}{s}\, e^{3/s}$.
5. $y(t) = \frac{3}{16}\, e^{5t} - \frac{1}{16}\, e^{-3t} - \frac{1}{8}\, e^t$.
6. $y(t) = 3\cos t + (k+1)\sin t$ so $k = 3$.

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Math 244 **Fourth Exam** **Spring 2011**

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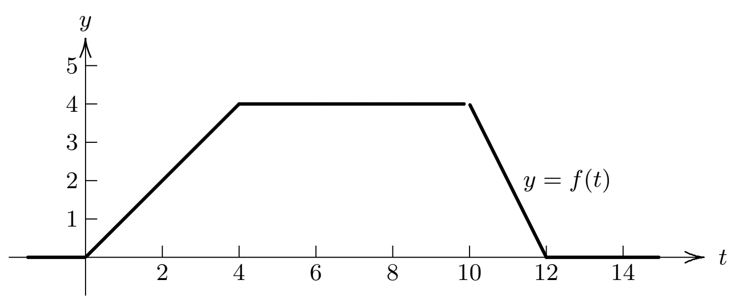
1. Find the Laplace transform of the function f defined by $f(t) = 2t^3 + e^t - 4\cos(3t)$. Show your steps clearly, pay careful attention to notation, and give the domain of the transform.
2. Suppose that $Y(s)$ is the Laplace transform of a function $y(t)$. Find $y(t)$ if $Y(s) = \dfrac{2s+3}{s^2+6s+34}$. You need to be certain that all of your steps are written carefully.
3. Use the method of Laplace transforms to solve the initial value problem
$$y'' - 3y' - 10y = u_2(t), \quad y(0) = 1, \quad y'(0) = 0.$$
Make certain that you define the functions that appear in your work.
4. Find the Laplace transform for the function f whose graph is given below.



You should clearly show your formula for $f(t)$.

5. Solve the initial value problem $y'' + 4y = 2\,\delta\left(t - \frac{\pi}{4}\right) + 6\,\delta(t - 4\pi)$, $y(0) = 0$, $y'(0) = 0$. Then (for 4 of the 12 points) find the amplitude of the steady state solution for $t > 4\pi$. In case you need them, note that $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \sin\beta\cos\alpha$ and $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$.

1. $\mathcal{L}\{f(t)\}(s) = \dfrac{12}{s^4} + \dfrac{1}{s-1} - \dfrac{4s}{s^2+9}$, $s > 1$.
2. $y(t) = 2e^{-3t}\cos(5t) - \dfrac{3}{5}\,e^{-3t}\sin(5t)$.
3. $y(t) = \dfrac{2}{7}\,e^{5t} + \dfrac{5}{7}\,e^{-2t} + u_2(t)h(t-2)$, where $h(t) = -\dfrac{1}{10} + \dfrac{1}{35}\,e^{5t} + \dfrac{1}{14}\,e^{-2t}$.
4. $\mathcal{L}\{f(t)\}(s) = \dfrac{1}{s^2}\left(1 - e^{-4s} - 2e^{-10s} + 2e^{-12s}\right)$.
5. The solution is $y(t) = u_{\pi/4}(t)\sin(2t - \pi/2) + 3u_{4\pi}(t)\sin(2t - 8\pi)$. For $t > 4\pi$, this function simplifies to $y(t) = -\cos(2t) + 3\sin(2t)$ so the amplitude is $\sqrt{10}$.