Math 244 **Differential Equations** **Fall 2016**

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A differential equation is an equation that involves the derivatives of some function. The usual goal is to find a function that satisfies the differential equation. For example, let y represent some function of x and consider the differential equation $y'' + 2y' - 3y = -9x$, where the primes indicate differentiation with respect to x. For a novice, it is not at all clear how to go about finding a function y that satisfies this equation. However, it is easy to check whether or not a given function satisfies the equation. Suppose that the function y is given by $y = e^{-3x} + 3x + 2$. Then $y' = -3e^{-3x} + 3$ and $y'' = 9e^{-3x}$. Substituting the function y and its derivatives into the differential equation yields

$$\begin{aligned} y'' + 2y' - 3y &= 9e^{-3x} + 2\big(-3e^{-3x} + 3\big) - 3\big(e^{-3x} + 3x + 2\big) \\ &= 9e^{-3x} - 6e^{-3x} + 6 - 3e^{-3x} - 9x - 6 \\ &= -9x. \end{aligned}$$

This shows that the function $y = e^{-3x} + 3x + 2$ is a solution to the differential equation. These are easy calculations; coming up with the function y in the first place is a much more difficult problem. The purpose of this course is to develop methods for finding the function y from the differential equation.

Our textbook, *Elementary differential equations and boundary value problems, ninth edition* by Boyce and DiPrima (note that we will be using the 9th edition even though a 10th edition has appeared), is entirely devoted to the problem of solving differential equations. In spite of the size of the book, it only begins to scratch the surface of this area of mathematics. Differential equations arise in a number of physical situations and the need for a solution is a motivating force for solving such equations. However, the problems involved in differential equations are also interesting from a purely mathematical perspective and a number of mathematicians have studied them in great detail. We will look at some of the basic techniques and consider a few applications that hint at the usefulness of differential equations.

The textbook is, I believe, student friendly and I will be expecting you to do some reading on your own. We will discuss the basic ideas in class and work some problems. Homework problems will be assigned after almost every class period. It is highly recommended that you do all of the problems before the next class. I will only be collecting a few of the homework problems that are assigned. This means that you will need to discipline yourself to make sure you do ALL of the problems in a timely manner, not just the ones that will be graded. (For the record, not doing all of the assigned problems is one of several reasons why many students earn low grades on the first exam.) The problems to turn in and their due dates will be noted in advance. When grading your work, in addition to the validity of your solutions, I will also pay attention to your use of notation and the presentation of your answers (neatness, organization, complete sentence conclusion, etc.). A correct answer will not receive full credit if the line of reasoning leading up to it is incorrect or unclear.

There will be four in-class exams during the semester and a comprehensive final exam. The dates for the in-class exams are Sept. 23, Oct. 19, Nov. 7, and Dec. 7. Rescheduling of exams will be arranged only in rare circumstances and I need to be notified in advance if such a situation arises. Each of these exams will be worth 60 points while the comprehensive final exam will be worth 100 points. The scores on all of the collected homework problems during the semester will be converted to a 60 point scale. The grade for the course will be based on these 400 points.

Since I will be holding your written work to a higher standard than you may be accustomed to in your math classes, here are a few guidelines and a sample solution.

1. Unless explicitly given, copy the problem (or an abbreviated form of it), whether from the book, from the assignments page, or from a handout. If from the book, include the section of the book in which it is located and the problem number.
2. Present a careful solution that includes brief explanations, correct notation, and organized equations. Your conclusions should be stated in complete sentences using correct units when appropriate.
3. Check your solutions for accuracy and for quality of exposition, that is, proof and edit your work.

Problem: Suppose that a falling object satisfies the initial value problem $dv/dt = 9.8 - \frac{1}{4}v$ and $v(0) = 0$, where $v(t)$ is the velocity of the object in m/sec at time t seconds. (Note that positive velocity indicates that the object is falling.) Find the time required for the object to reach 90% of its limiting velocity and, assuming the object starts 500 meters above the ground, the distance traveled by the object during this time. In addition, determine the number of seconds it takes for the object to fall to the ground.

Solution: First, solve the initial value problem:

$$\frac{dv}{dt} = -\frac{1}{4}(v - 39.2) \quad \Rightarrow \quad \frac{d}{dt}(v - 39.2) = -\frac{1}{4}(v - 39.2) \quad \Rightarrow \quad v(t) - 39.2 = Ce^{-t/4}$$

$$v(0) = 0 \quad \Rightarrow \quad v(t) = 39.2(1 - e^{-t/4})$$

The limiting velocity is $\lim_{t\to\infty} v(t) = 39.2$ so we must solve $v(t) = 0.9 \cdot 39.2$:

$$1 - e^{-t/4} = 0.9 \quad \Rightarrow \quad e^{t/4} = 10 \quad \Rightarrow \quad t = 4\ln 10 \approx 9.210340.$$

Hence, the object reaches 90% of its limiting velocity after about 9.21 seconds.

Now let $f(t)$ be the distance in meters that the object has fallen after t seconds. Then f satisfies the differential equation

$$f'(t) = v(t) = 39.2(1 - e^{-t/4}), \quad f(0) = 0$$

and thus

$$f(t) = 39.2t + 156.8\,e^{-t/4} - 156.8,$$

which indicates that the object drops $f(4\ln 10) \approx 219.93$ meters during the time it takes to reach 90% of its limiting velocity.

Finally, we solve $f(t) = 500$ with a calculator (or Maple or WolframAlpha) to find the time required for the object to reach the ground:

$$39.2t + 156.8\,e^{-t/4} - 156.8 = 500 \quad \Rightarrow \quad t \approx 16.6935.$$

Therefore, it takes approximately 16.69 seconds for the object to fall to the ground. ■

There is no specific template for writing solutions and (within limits) you are free to develop your own style. However, for most of you, this method of presentation will require some extra work on your part. Since writing solutions in this way requires more time and effort, you might wonder why I am insisting that you do so. First of all, the ability to present technical material clearly is a valuable skill, and it takes practice to acquire this skill. Secondly, by writing reasons for your steps and using correct notation, you are forced to better understand the mathematical processes behind the steps. A third reason is that it is important in all walks of life to be able to communicate clearly and effectively; it usually takes more care and thought to do so with technical or abstract material. Finally, I want you to take pride in your work, both in terms of explanation/clarity and obtaining the correct answer. Please take these guidelines seriously as you write your solutions for the problems that you are turning in.