Math 126: Calculus II **Spring 2024**

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Calculus has its roots in two geometric problems: determining the areas of regions that have curved boundaries and finding tangents to curves other than circles. Using geometric techniques, the ancient Greeks made progress solving these two problems for the conic sections (circles, ellipses, parabolas, and hyperbolas) and some other easily defined curves. With the introduction of analytic geometry (the familiar x, y coordinate system, which provides a strong link between algebra and geometry) in the middle of the seventeenth century, a vast array of new curves were considered, and algebraic techniques were developed to determine areas and tangents for these curves. Not long after this development, Newton and Leibniz independently discovered that the problems of areas and tangents were related to one another in a simple way; this discovery is considered to be the birth of calculus. The fact that many problems in the physical sciences can be reduced to finding areas or tangents has made calculus one of the cornerstones of the scientific revolution.

Mathematics in general, and calculus in particular, is often taught as a set of algorithms and/or skills to be mastered and the focus is usually on one skill at a time. Several sample problems are explained then a large collection of similar problems are given as exercises. These exercises can often be solved by imitation rather than understanding. While skills are certainly important and an essential part of problem solving, other crucial aspects of problem solving include understanding the underlying concepts, knowing which skills to use, and combining several skills in a multi-step problem. Learning to solve nonroutine problems, those without an example to imitate and requiring multiple ideas and skills, is one of the goals of this class. These sorts of problems require more time and effort and can be frustrating, but the satisfaction of solving such a problem is much greater than simply cranking out an answer to a plug and chug problem. You will be given ample opportunity to experience such frustration and satisfaction during the next four months.

A typical class period will take the following form: a brief overview of previous material, including a discussion of some of the homework problems, followed by an interactive lecture on new material and, as time permits, a preview of what to expect on the next assignment. As an understanding of the material from the previous lecture is often needed in the next lecture, it is necessary to do the homework before the next class period; this may require some self-discipline on your part. There will usually be enough time in class to go over some of the homework problems but certainly not all of them. If you have further questions, seek assistance from other sources: fellow students taking the class, my office hours, various tutoring options, etc.

It is important that you come to class on time (even for early morning classes) and be ready to think as class begins. (This is the educational equivalent of stretching and warming up prior to an athletic performance.) In addition, be prepared to concentrate for an extended period of time, perhaps 45 minutes or so; I include this observation because attention spans have been compromised a bit by electronic devices and online formats. Also, be certain that your cell phones are turned off before entering the classroom. Finally, please refrain from eating in class; bringing your breakfast or a snack to class is not an option as it is a distraction for other students.

Homework problems will be assigned after every lecture and you should do them before the next class period; allow at least 2–3 hours for each assignment. It is important to realize that working on and struggling with problems is the best way to learn new material. We will go over some of the problems in class the next period, but you should seek extra help if you still have unresolved questions. I will only collect a few of the assigned homework problems (these will be noted in advance and these are the only problems I want turned in) and grade each assignment on a 12 point scale. In addition to the mathematical content of your homework, I will be checking for neatness and correct use of notation. Whenever relevant, you should use complete sentences in your homework solutions. I have high standards for written work and expect you to learn how to present technical material in a clear and concise manner (see the examples below). It is a good idea to recopy the solutions to the few problems that you are turning in; the solution should be somewhat polished as opposed to a recorded history of all your unsuccessful attempts.

The following two examples illustrate how homework solutions should be presented. First of all, if the problem is from the textbook, then you should copy the problem (or some abbreviated variation of the problem). Then give a full solution to the problem, using complete sentences when appropriate. For "purely mathematical" problems (such as the first one below), there may be no need to use sentences. Actually, an equation is a sentence, and this may be all that is required to present the solution to the problem. In other cases (see the second example), some technical writing, involving complete sentences and an explanation of the symbols and methods used, will be expected.

Problem 1: Find and simplify the derivative of the function $f(x) = \tan^3(4x)$.

Solution: Using the derivative formula for $\tan x$ and the Chain Rule, we find that

$$f'(x) = 3\tan^2(4x)\sec^2(4x) \cdot 4 = 12\tan^2(4x)\sec^2(4x).$$

Problem 2: Determine the interval(s) on which the function $g(x) = xe^{-2x}$ is increasing and those on which it is decreasing.

Solution A: The sign of the derivative of the function g indicates whether g is increasing (g' is positive) or decreasing (g' is negative). We can find the derivative of g by using the product rule:

$$g'(x) = x(-2)e^{-2x} + e^{-2x} = (1-2x)e^{-2x}.$$

Note that $g'(x) = 0$ only when $x = \frac{1}{2}$ and that e^{-2x} is always positive. Since g' is positive on the interval $(-\infty, \frac{1}{2})$, the function g is increasing on $(-\infty, \frac{1}{2}]$. Since g' is negative on the interval $(\frac{1}{2}, \infty)$, the function g is decreasing on $[\frac{1}{2}, \infty)$.

Solution B: We first need to find the values of x for which $g'(x) = 0$ or $g'(x)$ does not exist.

$$g(x) = xe^{-2x} \Rightarrow g'(x) = x(-2)e^{-2x} + e^{-2x} = (1-2x)e^{-2x};$$
$$g'(x) = 0 \Rightarrow x = \tfrac{1}{2}; \qquad g'(x) \text{ exists for all } x;$$
$$\text{for } -\infty < x < \tfrac{1}{2},\ g'(x) > 0; \quad \text{for } \tfrac{1}{2} < x < \infty,\ g'(x) < 0.$$

Therefore, the function g is increasing on the interval $(-\infty, \frac{1}{2}]$ and decreasing on the interval $[\frac{1}{2}, \infty)$.

Either form of the solution for the second problem is fine; pick a style with which you are most comfortable. The important points are to write a clear solution to the stated problem using explanations (which may be brief) and correct notation, and to finish the problem with a complete sentence (not a boxed number!) that summarizes your answer.

Homework is due at the beginning (PLEASE NOTE) of class and late homework will not typically be accepted. You should not be overly concerned about missing an occasional assignment as I usually discard one or two of the lowest homework scores during each portion of the course (the time period between exams), but you should turn in as many as possible. (If you know in advance that you will miss a class, have someone turn in your homework and borrow some notes on the material that you missed.) I do not make elaborate comments on the homework—enough to point out your error. Come in for assistance if you are still stuck after looking at the problem again or if you fail to understand why something has been marked as incorrect. Pay attention to your errors and learn from your mistakes, both computational and conceptual.

Two other comments about the homework are worth noting. First of all, you are on your own to do the problems that are not collected. You will need to be disciplined enough to do these problems. One of the more common reasons why students do poorly on an exam is failure to do enough homework problems. Secondly, pay careful attention to your calculations as I will not be giving a lot of partial credit. It does not take that much extra effort to do things correctly the first time; we all expect this of the people we hire to do work for us. As far as the course as a whole is concerned, I strongly encourage you to spend a few minutes reading (and thinking about) a section before we discuss it in class. This will not only help improve your understanding of the lecture, but it will also give you practice reading technical material.

There will be three in-class exams and each exam will be worth 60 points. The dates for these are Feb. 9, Mar. 8, and Apr. 26. Rescheduling of exams will be arranged only in rare circumstances and I need to be notified in advance if such a situation arises. The real check of what a person has learned from a math class is how much they know at the end of the course. Consequently, the final exam (scheduled for Monday morning May 13 for the 8 AM class and Tuesday morning May 14 for the 10 AM class) will be comprehensive and be worth 80 points. The total of all of the homework scores will be converted to a 60 point scale. The grade for the course will be based on these 320 points; I will keep you posted on the grading scale during the semester. Speaking of exams, there should not be a need to leave the classroom during a 55 minute exam. However, if it is necessary to do so, you must leave your phone on your desk or in your backpack.

Students often ask "Is Calculus II more difficult than Calculus I?" For most students, the answer is 'yes, it is definitely more difficult.' Some more prerequisite material from algebra and trigonometry is needed (such as an increased emphasis on $\ln x$ and the inverse trigonometric functions) as well as much of the material from Calculus I. The key concepts, integration and infinite series, are more abstract than the derivative, and the key skill, antidifferentiation, is harder than differentiation. In addition, some of the applications are more sophisticated and there is a greater emphasis on understanding than imitation. On the positive side, the ideas and applications in Calculus II are more interesting than those of Calculus I.

Calculus is an abstract subject; time and effort are necessary to get a good grasp of its content. A fair amount of prerequisite knowledge, including but not limited to algebra and trigonometry, is required. The key concepts must be thought about carefully and understood before the mechanics make sense. In addition (as you may be aware), I have rather high standards and expectations for my students, but I am available and willing to help you meet these expectations. Since all of this may sound rather intimidating, here are some guidelines. You may want to refer to these more than once during the semester.

(1) You have been studying math for twelve or more years and have thus developed some habits and patterns to help you succeed in these classes. Many of these habits involve practicing skills, imitating solutions, memorizing formulas, and perhaps cramming for exams. To succeed in this class, you may need to begin to break some of these habits. As doing so may be rather difficult, you will need to make a concerted and conscious effort. I will be asking you (somewhat regularly) to understand the reasoning behind the skills, to solve problems without an example to imitate, to derive formulas rather than memorize them, and to build a framework of understanding so that you do not need to cram for exams.

(2) Study the book (and the extra notes) carefully. The sections are short enough that you should be able to read and understand every detail. Think hard about the ideas and ask questions on anything that is not clear. You will be expected to do some thinking and reading on your own and to work on some problems without seeing an example first. Don't wait a week to seek help if you start having trouble.

(3) Keep up with the homework. In a class such as this, you cannot get by with studying every once in a while or waiting until a few days before the test. Note that reading solutions and solving problems are different skills. Learning mathematics is an active process; observing others solve problems is not sufficient. Learn the techniques, don't just imitate examples. Be certain you understand the main ideas. If you choose to work with others, make sure that you are actively participating, especially paying attention to how to start problems on your own.

(4) Pay attention to notation and presentation. An inability to use correct notation is often a sign that the concepts have not yet been learned. Go back over the guidelines for writing your turned-in homework solutions and take them seriously. In particular, remember to recopy the problem (when appropriate) and to use a legitimate complete sentence to finish the problem. When solving a problem, do your best to stay focused so as not to make "stupid" mistakes. An example of such a mistake is writing $4 \cdot 2 = 6$. These errors come from a lack of concentration and we can do our best to make fewer of them by paying more attention to our work. Whenever any of us is learning new things, we are bound to make mistakes that are neither trivial nor silly. Rather than chastising yourself for being incompetent, you can learn from these higher level mistakes and improve your understanding of the relevant concept or skill.

(5) Use discretion while taking notes during class. You need not write down everything that appears on the board. In fact, this is usually a poor strategy for taking notes in a math class. It is better to watch and think rather than mindlessly transcribe. You should also be aware that I say some important things without ever writing them on the board.

(6) Learning mathematics requires extended periods of distraction-free thought. Given the prevalence of technology in your lives, you will need to make a concerted effort to "unplug" your mind and focus on the task at hand. This sort of mindfulness will increase your understanding, help you avoid careless errors, and perhaps reduce the stress in your life.

In order to make them explicit, here are the goals for this course.

- to develop quantitative reasoning skills;
- to learn how to read technical material; [reading the textbook and notes on your own]
- to learn to write technical information correctly and clearly; [via feedback on collected HW problems]
- to take pride in your work and to avoid errors; [see item (4) above]
- to learn how to solve nonroutine problems; [see the second paragraph of the syllabus]
- to appreciate/understand how mathematicians view mathematics; [comments made during lectures]
- to comprehend some aspects of calculus.

Some of these may not be goals you envisioned for this class, but I encourage you to view the class in this light. Many of these goals will serve you well in the years ahead regardless of your life path.

If you are a student with a disability who needs accommodations for this course, please contact Richard Middleton-Kaplan (Director of Academic Support Services, office: Olin 314, middletr@whitman.edu) for help developing a plan to address your academic needs. All information about disabilities is private. If the Academic Resource Center notifies me that you are eligible to receive an accommodation due to a verified disability, I will do my best to provide that accommodation in a discreet manner. If your accommodation includes special exam arrangements, please contact me several days before the exam.

In accordance with the College's Religious Accommodations Policy (see link below), I will provide reasonable accommodations for all students who, because of religious observances, may have conflicts with scheduled exams, assignments, or required attendance in class. Please review the course schedule at the beginning of the semester to determine any such potential conflicts and let me know if such a situation arises; I will do my best to provide such accommodations. If you believe that I have failed to abide by this policy, a link to the Grievance Policy is given below.

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