Math 225: Calculus III **Spring 2014**

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First year calculus is concerned with the differentiation and integration of functions of a single variable. However, many physical properties depend on more than one variable. For example, the volume of a cone is a function of both its radius and its height. As another example, the temperature at various places in a room is a function of three variables—the three variables required to locate the particular point in the room. This function can be represented by $T(x, y, z)$, where (x, y, z) gives the location of a point in the room. Second year calculus extends the ideas of differentiation and integration to functions of more than one variable. As we will see, the material becomes more relevant and more complicated at the same time.

Calculus is part of a branch of mathematics known as real analysis. This subject is one of the crowning achievements of the human intellect. It illustrates a desire to understand the physical world as well as the pursuit of knowledge for the sake of curiosity and beauty. Real analysis in general and calculus in particular are challenging courses. A substantial background of mathematical knowledge and a willingness to think deeply are required to study this subject. Here are some guidelines that may be of help to you.

1. Study the book (J. Stewart, *Calculus, Early Transcendentals*, 7th ed.) carefully, think hard about the ideas, and ask questions. You will be expected to read the book (remember that reading a math book is slow going and often requires pencil and paper) and to work on some problems without seeing an example first. (For the record, we will be covering most of Chapters 10, 12, 14, 15, and 16.)
2. Keep up with the homework. In a class such as this you cannot get by with studying every week or so. If you start having trouble, get help as soon as possible. Since I will not be collecting homework every period, you will need to take the initiative to get all of the work done. Given the schedule for this class, plan on spending at least two hours doing calculus on Mondays and Thursdays.
3. Note that reading solutions and solving problems are different skills. Learning mathematics is an active process; observing others solve problems is not sufficient. Learn the techniques, don't just imitate examples. Be certain you understand the main ideas. If you choose to work with others, make sure that you are actively participating, especially paying attention to how to start problems on your own.
4. Pay attention to your use of notation and style of presentation. On homework and exams, I will be checking for correct use of notation and clarity of explanation as well as the use of procedures. A correct answer in a box surrounded by unreadable stuff will not receive any credit. An inability to use correct notation is often a sign that the concepts have not yet been learned. In addition, learning how to express technical material in a coherent manner is a valuable skill to acquire.
5. Use some discretion while taking notes. You need not write down everything that appears on the board. In fact, this is usually a poor strategy for taking notes in a math class. It is better to watch and think rather than blindly transcribe from the board. On the other hand, you should be aware that I say some important things without ever writing them on the board.

Homework problems will be assigned after almost every class period. You really should work on these before the next class period since new material builds on previous ideas. Allow at least 2–3 hours for each assignment. If time permits, we will go over some of the problems in class the next period. I will be collecting only a few problems a couple of times a week, usually on Mondays and Thursdays. Homework is due at the beginning of class and, except for legitimate excuses from the dean, late homework will not be accepted. To allow for some flexibility in your work schedule, I will drop some of the lower homework scores. For instance, if there end up being 6 assignments before an exam, I will only count the top 5 of the scores. Since I will be collecting only a few of the many assigned problems, you will need to discipline yourself in order to do them all. I may also give a quiz on occasion (with advance notice) and count this as another homework assignment.

There will be four in-class exams. The dates for these are Feb. 7, Mar. 7, Apr. 11, and May 6 (the first three are Fridays and the last one is a Tuesday). Rescheduling of exams will be arranged only in rare circumstances and I need to be notified in advance if such a situation arises. Each exam is worth 60 points. The real check of what a person has learned from a math class is how much they know at the end of the course. Consequently, the final exam will be comprehensive and count 100 points. The total of all of the homework/quiz scores will be converted to a 60 point scale. The grade for the course will be based on these 400 points. I do not grade on a curve, that is, I do not have a predetermined percentage of A's, B's, and so on. Each exam will have a grading scale based upon my sense of its difficulty and the actual performance of the class. Putting all of the exam scales together along with a range for the homework/quiz scores will give the scale for the entire course.

Since your writing for this class will be held to a higher standard than you are accustomed to, let me give you two examples. The idea is to move away from simply writing a bunch of equations and circling the final answer. You should work toward using complete sentences and adding some explanation.

Problem: Find the volume of the solid that is generated when the region under the curve $y = \sin x$ and above the x-axis on the interval $[0, \pi]$ is revolved around the x-axis.

Solution: Using the disk method, we find that the volume V is given by $V = \int_0^\pi \pi(\sin x)^2\,dx$. The half-angle formula for sine [which you need to know] allows us to evaluate the integral:

$$\pi \int_0^\pi \sin^2 x\,dx = \frac{\pi}{2}\int_0^\pi \big(1-\cos 2x\big)\,dx = \frac{\pi}{2}\Big(x - \frac{1}{2}\sin 2x\Big)\Big|_0^\pi = \frac{\pi^2}{2}.$$

The volume generated is thus $\pi^2/2$ cubic units.

Problem: Evaluate $\displaystyle\int_0^1 \frac{x^3}{\sqrt{1-x^2}}\,dx$. (You may gloss over the fact that this is an improper integral.)

Solution: The trig substitution $x = \sin\theta$ transforms the integral into an easier form:

$$\begin{aligned}
\int_0^1 \frac{x^3}{\sqrt{1-x^2}}\,dx &= \int_0^{\pi/2} \frac{\sin^3\theta}{\cos\theta}\cos\theta\,d\theta && \text{letting } x = \sin\theta \\
&= \int_0^{\pi/2} \big(1-\cos^2\theta\big)\sin\theta\,d\theta && \text{using } \sin^2\theta = 1-\cos^2\theta \\
&= \Big(-\cos\theta + \frac{1}{3}\cos^3\theta\Big)\Big|_0^{\pi/2} && \text{find an antiderivative} \\
&= 0 - \Big(-1 + \frac{1}{3}\Big) = \frac{2}{3} && \text{plug in endpoints}
\end{aligned}$$

The value of the integral is thus $2/3$.