Total: Name:

Math 125 **Second Exam** **Fall 2016**

Write neat, concise, and accurate solutions to each of the problems below—I will not give partial credit for steps I cannot follow. Include all relevant details, use correct notation, and finish problems with a complete sentence when appropriate. No electronic devices are allowed for this exam. Each of the first eight problems is worth 7 points and the last problem is worth 4 points.

1. Carefully state the definition of the derivative of a function f.

2. Find and simplify the derivative of the function f defined by $f(x) = e^{-3x}\sin(2x)$.

3. Find an equation for the line tangent to the graph of $y = x^2 - \ln(4x - 3)$ when $x = 1$.

4. Find the maximum and minimum outputs of $f(x) = \dfrac{\sin x}{2 + \cos x}$ on the interval $[0, \pi]$.

5. Consider the function f defined by $f(x) = -2x^3 + 12x^2 + 30x$. Use the first derivative test to show that f has a relative maximum value at some point x_0, then find the (x, y) coordinates of the point on the graph that corresponds to this value.

6. Suppose that $f'(x) = \dfrac{x^3 - 12x^2}{x - 6}$, where f is some function whose domain is all real numbers except $x = 6$. (You are not expected to find the function f.) Find the intervals on which the function f is increasing and those on which it is decreasing. Make careful note as to whether or not the endpoints are included in the intervals.

7. Suppose that g is a function that satisfies $g'(t) = -k\big(g(t) - 20\big)$, $g(0) = 90$, $g(2) = 80$, where k is some positive constant. Find the value of k.

8. Let C be the circle of radius 6 centered at the origin. Find the (x, y) coordinates of a point on C for which the quantity x^2y has a maximum value.

9. This problem has two short parts.

   a) Consider the continuous function f defined $f(x) = e^x \tan x$. Since $f(0) = 0$ and $f(1) \approx 4.23$, it follows that the equation $e^x \tan x = 2$ has a solution on the interval $[0, 1]$. What theorem guarantees this fact?

   b) Write down the derivative formulas for $\tan x$ and $\arctan x$.

end of previous exam: only problems 1, 4, 5, 6, 8, 9a, and half of 9b are relevant for our exam.

Here are some other possible questions for the material that we have covered.

1. Let $f(x) = x - 3x^{2/3}$. Determine the intervals on which f is increasing and those on which it is decreasing.

2. Let $f(x) = b^2x - x^3$, where b is a positive constant. Determine the intervals on which f is increasing and those on which it is decreasing.

3. Find the exact value of all the trigonometric functions given that $\tan x = -4$ and $\pi/2 < x < \pi$.

4. Find a cubic polynomial that has a relative minimum output when $x = -2$ and a relative maximum output when $x = 5$.

5. Find the maximum and minimum outputs of the function $f(x) = 4x + \dfrac{3}{3x-1}$ on the interval $[0.4, 2]$.

6. Determine the nature of all the critical inputs for the function $f(x) = x - 4\sqrt{x}$.

7. Find the volume of the largest right circular cylinder that can be inscribed in a sphere of radius r. What fraction of the sphere is occupied by the optimal cylinder?

8. Find and simplify the derivative of $f(x) = x^2 \sin(2x)$.

9. Find and simplify the derivative of $G(t) = \sec^2(5t)$.

10. Find an equation for the line tangent to the curve $y = 2\sin x - 3\cos 2x$ when $x = \pi/6$.

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1. Evaluate $\lim_{x \to 9} \dfrac{x^2 - 10x + 9}{\sqrt{x} - 3}$.

2. Find and simplify the derivative of the function h defined by $h(x) = \sqrt[3]{3x^2 - 9x + 10}$.

3. Find an equation for the line tangent to the graph of $y = x + \dfrac{6}{x}$ at the point when $x = 2$.

4. Give one example for each; no explanation is required.

   a) a rational number between 4 and 5

   b) a rational function f that is not continuous at the points $x = 2$ and $x = 3$

   c) a continuous function g that is not differentiable at $x = -2$

5. Suppose that the height h in feet of a beanstalk after t hours is $h = t^3 + 6t^2 + 30t$. When is the rate of growth of the beanstalk 210 feet per hour?

6. Find all of the values of x for which the graph of the function f defined by $f(x) = (5x + 8)^2(4x - 15)^5$ has a horizontal tangent line.

7. Carefully state the definition of the derivative, then give a one sentence explanation concerning what it tells you about the graph of a function.

8. Use the definition of the derivative to find the derivative of the function f defined by $f(x) = x^2 - 3x$.

9. Find and simplify the derivative of the function f defined by $f(x) = \dfrac{x+1}{x^2+2x+3}$.

10. Consider the curve defined by the equation $y = ax^2 + b$, where a and b are constants. Suppose that the point $(1, 5)$ is on this curve and that the tangent line to the graph at this point goes through the point $(7, 1)$. Find the values of a and b.