I certify that I followed all of the posted guidelines for this exam. *Signature*:

Math 126 **Second Exam** **Fall 2020**

Write neat, concise, and accurate solutions to each of the problems below—I will not give partial credit for steps I cannot follow. Include all relevant steps, use correct notation, give sufficient details, and finish problems with a complete sentence; either in words or with an appropriate equation. Point values for each problem are indicated next to the problem. If at all possible, keep your solutions to six pages or less.

1. (4 points) State the definition of the derivative, including all of the appropriate words and symbols.

The derivative of a function f is another function f' defined by

$$f'(x) = \lim_{r \to x} \frac{f(r) - f(x)}{r - x}$$

for all values of x in the domain of f for which the limit exists.

2. (4 points) State the definition of the integral, including all of the appropriate words and symbols.

The integral of a continuous function f on an interval $[a, b]$, denoted $\int_a^b f(x)\,dx$, is defined by

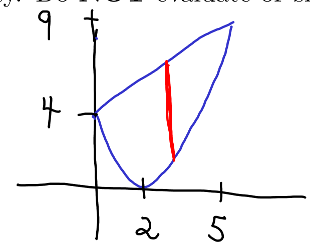
$$\int_a^b f(x)\,dx = \lim_{n \to \infty} \sum_{i=1}^{n} f\left(a + i \cdot \frac{b-a}{n}\right) \frac{b-a}{n}.$$

3. (5 points) Evaluate $\displaystyle\int \frac{4x+5}{x^2+16}\,dx$.

$$\int \frac{4x+5}{x^2+16}\,dx = \int \left(\frac{4x}{x^2+16} + \frac{5}{x^2+16} \right) dx$$

$$= 2\ln(x^2+16) + \frac{5}{4}\arctan\left(\frac{x}{4}\right) + C$$

Let R be the region that lies between the curves $y = x + 4$ and $y = (x-2)^2$. For problems 4–9, write an expression involving integrals that represents the requested quantity. Do **NOT** evaluate or simplify the integrals. Include a sketch of R to the right of the space below.



4. (4 points) the area of R

$$A = \int_0^5 \left((x+4) - (x-2)^2\right) dx$$

5. (4 points) the volume of the solid that is generated when R is revolved around the x-axis

$$V = \int_0^5 \left(\pi (x+4)^2 - \pi (x-2)^4\right) dx$$

6. (4 points) the volume of the solid that is generated when R is revolved around the y-axis

$$V = \int_0^5 2\pi x \left(x + 4 - (x-2)^2\right) dx$$

7. (4 points) the volume of the solid that is generated when R is revolved around the line $x = 7$

$$V = \int_0^5 2\pi (7 - x)\left(x + 4 - (x-2)^2\right) dx$$

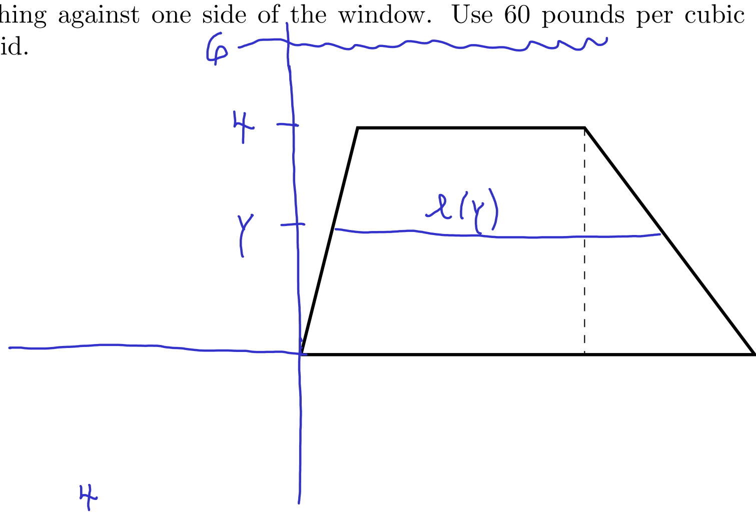
8. (4 points) the volume of the solid whose base is R and each cross-section of the solid taken perpendicular to the x-axis is a semicircle

$$V = \int_0^5 \frac{\pi}{2}\left(\frac{x + 4 - (x-2)^2}{2}\right)^2 dx$$

9. (4 points) the x-coordinate of the center of mass of R use ρ for density

$$\bar{x} = \frac{\int_0^5 x \rho \left(x + 4 - (x-2)^2\right) dx}{\int_0^5 \rho \left(x + 4 - (x-2)^2\right) dx}$$

10. (6 points) The figure below represents a vertically submerged window in the shape of a trapezoid. The distance across the top is four feet, the distance across the bottom is eight feet, the height is four feet, and the top of the window is two feet below the surface of the liquid. Find the total force of the liquid pushing against one side of the window. Use 60 pounds per cubic foot for the weight density of the liquid.



$\ell(0) = 8$

$\ell(4) = 4$

$\ell(y) = 8 - y$

$$F = \int_0^4 60(6-y)(8-y)\,dy$$

$$= 60\int_0^4 (y^2 - 14y + 48)\,dy$$

$$= 60\left(\frac{1}{3}y^3 - 7y^2 + 48y\right)\Big|_0^4$$

$$= 60\left(\frac{64}{3} - 112 + 192\right)$$

$$= 60\left(\frac{64}{3} + 80\right)$$

$$= 20 \cdot 64 + 4800$$

$$= 6080$$

The total force on the window is 6080 pounds.

11. (6 points) Evaluate $\displaystyle\int_2^\infty \frac{4}{x^2+6x+8}\,dx$.

$$\frac{4}{x^2+6x+8} = \frac{4}{(x+2)(x+4)} = \frac{A}{x+2} + \frac{B}{x+4}$$

$$4 = A(x+4) + B(x+2)$$

$$x=-2 \implies 4 = 2A \implies A = 2$$
$$x=-4 \implies 4 = -2B \implies B = -2$$

$$\int \frac{4}{x^2+6x+8}\,dx = \int \left(\frac{2}{x+2} - \frac{2}{x+4}\right) dx$$
$$= 2\ln|x+2| - 2\ln|x+4| + C$$
$$= 2\ln\left|\frac{x+2}{x+4}\right| + C$$

$$\int_2^\infty \frac{4}{x^2+6x+8}\,dx = \lim_{b\to\infty} \int_2^b \frac{4}{x^2+6x+8}\,dx$$
$$= \lim_{b\to\infty} 2\ln\left|\frac{x+2}{x+4}\right| \Bigg|_2^b$$
$$= \lim_{b\to\infty} \left(2\ln\left|\frac{b+2}{b+4}\right| - 2\ln\frac{4}{6}\right)$$
$$= 2\ln 1 - 2\ln\frac{2}{3}$$
$$= 2\ln\frac{3}{2}.$$

12. (6 points) Find the length of the curve $y = 4x^{3/2}$ on the interval $[0, 10]$.
(Making use of the equation $a^3 - 1 = (a-1)(a^2+a+1)$ near the end of the evaluation of the integral should make the numbers come out nice and easy.)

$$y = 4x^{3/2}$$

$$\frac{dy}{dx} = 6x^{1/2}$$

$$L = \int_0^{10} \sqrt{1 + 36x}\, dx$$

$$= \frac{2}{3} \cdot \frac{1}{36} \left(1 + 36x\right)^{3/2} \Big|_0^{10}$$

$$= \frac{1}{54}\left(361^{3/2} - 1\right)$$

$$= \frac{1}{54}\left(19^3 - 1\right)$$

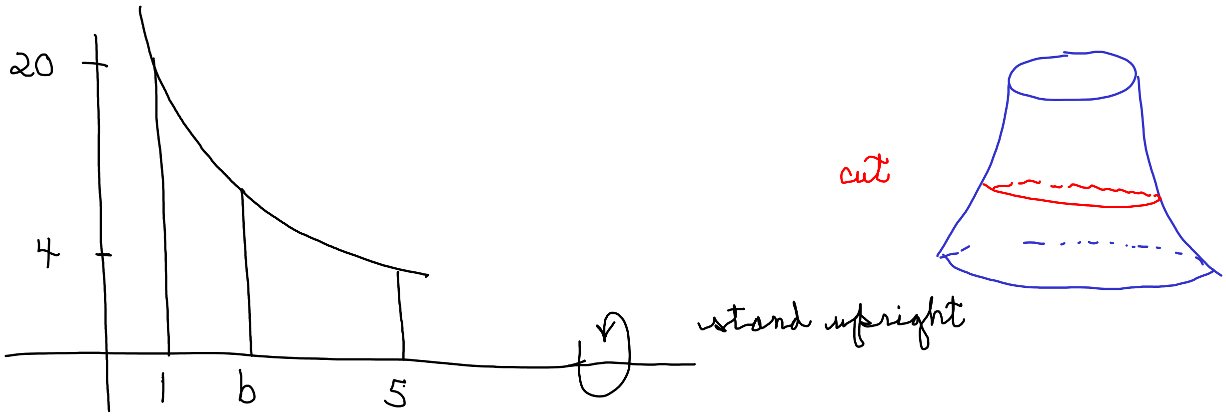
$$= \frac{1}{54}(19-1)(19^2 + 19 + 1)$$

$$= \frac{1}{3} \cdot 381$$

$$= 127$$

The length of the curve is 127 units.

13. (5 points) Let R be the region that lies below the curve $y = 20/x$ and above the x-axis on the interval $[1, 5]$ and let S be the solid that is generated when R is revolved around the x-axis. The solid S is cut into two pieces along the line $x = b$. Find the value of b so that the two pieces have the same volume.



b must satisfy the equation

$$\int_1^b \pi\left(\frac{20}{x}\right)^2 dx = \frac{1}{2}\int_1^5 \pi\left(\frac{20}{x}\right)^2 dx$$

$$\int_1^b \frac{1}{x^2}\,dx = \frac{1}{2}\int_1^5 \frac{1}{x^2}\,dx$$

$$-\frac{1}{x}\Big|_1^b = \frac{1}{2}\left(-\frac{1}{x}\Big|_1^5\right)$$

$$-\frac{1}{b} + 1 = \frac{1}{2}\left(-\frac{1}{5} + 1\right) = \frac{2}{5}$$

$$1 - \frac{2}{5} = \frac{1}{b}$$

$$b = \frac{5}{3}$$

The value of b is $\frac{5}{3}$.