Total: Name:

Math 126 **First Exam** **Spring 2011**

Write neat, concise, and accurate solutions to each of the problems below—I will not give partial credit for steps I cannot follow. Include all relevant steps, use correct notation, give sufficient details, and finish problems with a complete sentence when appropriate. Each problem is worth 6 points. No calculators are allowed for this exam.

1. State the definition of the integral. Be certain to include all of the appropriate words and symbols.
2. For the function f defined by $f(x) = \displaystyle\int_2^{3x} \tan(t^2)\,dt$, find $f'(x)$.
3. Evaluate $\displaystyle\int \sin(2x)\cos^3(2x)\,dx$.
4. Evaluate $\displaystyle\int 4x\sqrt{2x+1}\,dx$.
5. Evaluate $\displaystyle\int_{-4}^{4} 5\sqrt{16-x^2}\,dx$.
6. Evaluate $\displaystyle\int_0^1 \frac{x^2+2}{x^3+6x+1}\,dx$.
7. Evaluate $\displaystyle\int_1^{\infty} \frac{x}{(x^2+4)^3}\,dx$.
8. Evaluate $\displaystyle\int 4xe^{x/2}\,dx$.
9. Suppose that $v(t) = t^3 - 4t$ gives the velocity in meters per second of a particle at time t seconds. Find the distance traveled by the particle during the time interval $0 \le t \le 3$.
10. Find the area of the region bounded by the graphs of the equations $y = x^4$ and $y = 8x$.

Here are some other questions that could appear.

1. State the definition of the derivative. (Include all of the words.)
2. Carefully state the version (and only this version) of the Fundamental Theorem of Calculus that involves differentiating an integral. Note that I am NOT requesting a proof of this result.
3. By using an appropriate function (your function should not be a constant), find an estimate for the value of the integral $\displaystyle\int_2^4 \frac{1}{\sqrt{x^4-1}}\,dx$. After obtaining your estimate, explain (without evaluating the integral!) whether your estimate is larger or smaller than the actual value of the integral.
4. For each pair of integrals, determine which integral has the larger value. (You should not evaluate any integrals for this problem!) Be certain to justify your answers with a clear statement.

   a) $\displaystyle\int_4^{10} \sqrt{x^8+3}\,dx, \quad \int_4^{10} x^4\,dx$

   b) $\displaystyle\int_0^1 \cos^3 x\,dx, \quad \int_0^1 \cos^5 x\,dx$

Total: Name:

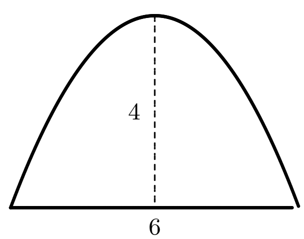
Math 126 **Second Exam** **Spring 2011**

Write neat, concise, and accurate solutions to each of the problems below—I will not give partial credit for steps I cannot follow. Include all relevant details, use correct notation, provide sufficient explanations, and finish problems with a complete sentence when appropriate. Each problem is worth 5 points. Calculators are not allowed for this exam.

1. State the definition of the integral (including all of the words).
2. Evaluate $\displaystyle\int \frac{8x-15}{x^2+25}\,dx$.
3. Evaluate $\displaystyle\int \frac{4x+7}{2x-1}\,dx$.

Let R be the region that lies between the curves $y = 2x$ and $y = x^2/4$. For problems 4–8, write down an expression involving an integral that represents the requested quantity. Do **NOT** evaluate the integrals.

4. the volume generated when R is revolved around the x-axis
5. the volume generated when R is revolved around the y-axis
6. the volume generated when R is revolved around the line $x = 10$
7. the volume of the solid whose base is R and each cross-section of the solid taken perpendicular to the y-axis is a square
8. the perimeter of R
9. Find the length of the curve $y = \dfrac{1}{12}x^3 + \dfrac{1}{x}$ on the interval $[1, 2]$.
10. Set up an integral (you do not need to evaluate it) that gives the force exerted by a liquid on one side of the vertically submerged parabolic plate shown below. The units on the figure are feet and the top of the plate is eight feet beneath the surface of the liquid. Assume that the weight density of the liquid is 60 lb/ft^3.



11. Evaluate $\displaystyle\int \frac{x-1}{x^2+x-6}\,dx$.
12. Evaluate $\displaystyle\int_0^1 \frac{6x^2}{(4-x^2)^{5/2}}\,dx$.

Total: Name:

Math 126 **Third Exam** **Spring 2011**

Write neat, concise, and accurate solutions to each of the problems below—I will not give partial credit for steps I cannot follow. Include all relevant details and use correct notation. Calculators are not allowed for this exam. Each problem is worth six points.

1. Give an example (use formulas, not patterns) with the indicated properties. Be certain to use correct notation. No explanation is required but your example should be elementary.

   a) a convergent sequence that is not monotone

   b) a monotone sequence that does not converge

   c) a conditionally convergent series

2. Find the sum of the series $\displaystyle\sum_{k=1}^{\infty} \frac{3^{k-1} - 2^k}{5^k}$.

For problems 3–6, determine whether or not the given series converges. You must provide clear justification along with complete details for your conclusion. Be certain to mention which test you are using and pay careful attention to your notation.

3. $\displaystyle\sum_{k=1}^{\infty} \frac{2k+1}{k^3+4k+5}$

4. $\displaystyle\sum_{k=1}^{\infty} \frac{4^k\, k!}{2 \cdot 7 \cdot 12 \cdot \cdots \cdot (5k+2)}$

5. $\displaystyle\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{7k+4}$

6. $\displaystyle\sum_{k=1}^{\infty} \frac{k}{\sqrt{2k^2+3k-1}}$

7. Find the limit of the sequence $\left\{\left(\dfrac{3n}{3n+1}\right)^n\right\}$. You must show how you arrived at your answer.

8. For each positive integer n, let $x_n = \dfrac{1}{n+1} + \dfrac{1}{n+2} + \dfrac{1}{n+3} + \cdots + \dfrac{1}{10n}$. Prove that the sequence $\{x_n\}$ is bounded.

9. Find the radius of convergence of the power series $\displaystyle\sum_{k=0}^{\infty} \frac{k^2}{3^k}\,(x+2)^k$

10. Let $\{f_n\}$ denote the Fibonacci sequence. Use the Principle of Mathematical Induction to prove that $\displaystyle\sum_{i=1}^{2n+1} (-1)^{i-1} f_i = f_{2n} + 1$ for each positive integer n.

Total: Name:

Math 126 **Final Exam** **Spring 2011**

Write neat, concise, and accurate solutions to each of the problems in the space provided—I will not give any credit for steps I cannot follow. Your solutions should be written in the style expected for collected homework problems. Pay particular attention to correct use of notation and use sentences when appropriate. Each of the thirteen problems is worth six points. You may use a calculator for this exam. However, I am assuming that you do not use your calculator or other electronic device for integrating functions (either definite or indefinite), summing infinite series, calling for help, etc; any such use will be treated as cheating.

1. State the definition of the derivative and the definition of the integral. Be certain to include all of the appropriate words.

2. Suppose that $v(t) = 9t - t^3$ gives the velocity in meters per second of a particle at time t seconds. Find the total distance traveled by the particle from time $t = 0$ to time $t = 4$ seconds.

3. Find all of the antiderivatives of the function $f(x) = 12x^2 \ln x$.

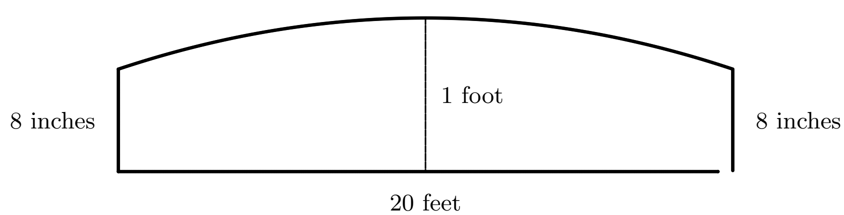
4. Find the sum of the series $\displaystyle\sum_{k=1}^{\infty} \frac{2^k + (-1)^{k+1}}{5^k}$.

5. Determine the interval of convergence for the power series $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k}{3^k(4k+1)}(x-5)^k$. Be certain that all of your conclusions are justified.

6. Consider the function f defined by $f(x) = \dfrac{\sin x - x}{x^2}$. Find the Maclaurin series for this function then use it to find the value of $f^{(101)}(0)$.

7. Use mathematical induction to prove that
$$f_3 + f_6 + f_9 + \cdots + f_{3n} = \frac{f_{3n+2} - 1}{2}$$
for each positive integer n, where f_n represents the nth Fibonacci number.

8. Determine (to the nearest cubic yard) the number of cubic yards of crushed rock necessary to make a roadbed one mile long with cross section (not to scale) shown below.



Assume that the crown of the roadbed is a parabola. (There are 5280 feet in a mile.)

9. Evaluate each of the following integrals.

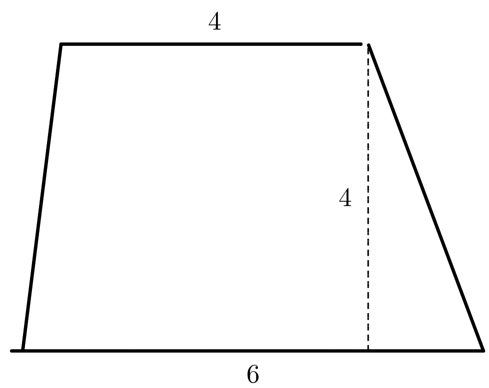
a. $\displaystyle\int \frac{x}{\sqrt{4-x^2}}\,dx$

b. $\displaystyle\int \frac{x}{4-x^2}\,dx$

c. $\displaystyle\int \frac{1}{\sqrt{4-x^2}}\,dx$

10. Evaluate $\displaystyle\int \frac{3x+2}{x^2-6x-16}\,dx$.

11. Find the force exerted by a liquid on one side of the vertically submerged trapezoidal plate shown below. The units on the figure are feet and the top of the plate is four feet beneath the surface of the liquid. Assume that the weight density of the liquid is 60 lb/ft^3.



12. Determine whether or not each of the following infinite series converges. Give reasons for your responses, including details and the names of any tests you use.

a. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt[k]{5}}$

b. $\displaystyle\sum_{k=1}^{\infty} \frac{k}{3^k}$

c. $\displaystyle\sum_{k=1}^{\infty} \frac{1}{7k+1}$

13. Let R be the region bounded by the curves $y = 6x - x^2$ and $y = x$. For each of the following, write down an expression involving one or more integrals that represents the requested quantity. **DO NOT EVALUATE THE INTEGRALS.**

a) the volume of the solid that is generated when R is revolved around the line $x = 10$

b) the volume of the solid whose base is R and each cross-section of the solid taken perpendicular to the x-axis is a semicircle

c) the x-coordinate of the center of mass of R, assuming that the density has a constant value of ρ

BONUS QUESTION: This is not part of the test. If you have extra time and want to continue thinking about mathematics, find the sum of the series $\displaystyle\sum_{k=1}^{\infty} \frac{k^2}{k!}$. A correct answer, along with coherent reasoning, is worth 3 extra points.