Preface

This book has been written in an attempt to provide a satisfactory textbook to be used as a basis for elementary courses in Non-Euclidean Geometry. The need for such a volume, definitely intended for classroom use and containing substantial lists of exercises, has been evident for some time. It is hoped that this one will meet the requirements of those instructors who have been teaching the subject regularly, and also that its appearance will encourage others to institute such courses.

The benefits and amenities of a formal study of Non-Euclidean Geometry are generally recognized. Not only is the subject matter itself valuable and intensely fascinating, well worth the time of any student of mathematics, but there is probably no elementary course which exhibits so clearly the nature and significance of geometry and, indeed, of mathematics in general. However, a mere cursory acquaintance with the subject will not do. One must follow its development at least a little way to see how things come out, and try his hand at demonstrating propositions under circumstances such that intuition no longer serves as a guide.

For teachers and prospective teachers of geometry in the secondary schools the study of Non-Euclidean Geometry is invaluable. Without it there is strong likelihood that they will not understand the real nature of the subject they are teaching and the import of its applications to the interpretation of physical space. Among the first books on Non-Euclidean Geometry to appear in English was one, scarcely more than a pamphlet, written in 1880 by G. Chrystal. Even at that early date the value of this study for those preparing to teach was recognized. In the preface to this little brochure, Chrystal expressed his desire to bring "pangeometrical speculations under the notice of those engaged in the teaching of geometry." He wrote: "It will not be supposed that I advocate the introduction of pangeometry as a school subject; it is for the teacher that I advocate such a study. It is a great mistake to suppose that it is sufficient for the teacher of an elementary subject to be just ahead of his pupils. No one can be a good elementary teacher who cannot handle his subject with the grasp of a master. Geometrical insight and wealth of geometrical ideas, either natural or acquired, are essential to a good teacher of geometry; and I know of no better way of cultivating them than by studying pangeometry."

Within recent years the number of American colleges and universities which offer courses in advanced Euclidean Geometry has increased rapidly. There is evidence that the quality of the teaching of geometry in our secondary schools has, accordingly, greatly improved. But advanced study in Euclidean Geometry is not the only requisite for the good teaching of Euclid. The study of Non-Euclidean Geometry takes its place beside it as an indispensable part of the training of a well-prepared teacher of high school geometry.

This book has been prepared primarily for students who have completed a course in calculus. However, although some mathematical maturity will be found helpful, much of it can be read profitably and with understanding by one who has completed a secondary school course in Euclidean Geometry. He need only omit Chapters V and VI, which make use of trigonometry and calculus, and the latter part of Chapter VII.

In Chapters II and III, the historical background of the subject has been treated quite fully. It has been said that no subject, when separated from its history, loses more than mathematics. This is particularly true of Non-Euclidean Geometry. The dramatic story of the efforts made throughout more than twenty centuries to prove Euclid's Parallel Postulate, culminating in the triumph of rationalism over tradition and the discovery of Non-Euclidean Geometry, is an integral part of the subject. It is an account of efforts doomed to failure, of efforts that fell short of the goal by only the scantiest margin, of errors, stupidity, discouragement and fear, and finally, of keen, penetrating insight which not only solved the particular problem, but opened up vast new and unsuspected fields of thought. It epitomizes the entire struggle of mankind for truth.

A large number of problems has been supplied—more than will be found in other books on this subject. The student will enjoy trying his hand at original exercises amid new surroundings and will find their solution a valuable discipline. But the problems are not merely practice material, they form an integral part of the book. Many important results are presented in the problems, and in some instances these results are referred to and even used.

It is believed that the material in the Appendix will be found helpful. Since most students in this country are not acquainted with the propositions of Euclid by number, we have reprinted the definitions, postulates, common notions and propositions of the First Book. An incidental contact with these propositions in their classical order may not be the least of the benefits to be derived from this study. Also included are sections on the hyperbolic functions, the theory of orthogonal circles, and inversion. They are sufficiently extensive to give the reader who has not previously encountered the concepts an adequate introduction. These topics, when introduced at all in other courses, are generally presented in abstract and isolated fashion. Here they are needed and used, and the student may possibly be impressed for the first time with their practical importance.

There will be those who will classify this book as another one of those works in which the authors "wish to build up certain clearly conceived geometrical systems, and are careless of the details of the foundations on which all is to rest." It is true that no attempt has been made here to lay a complete and thoroughly rigorous foundation for either the Hyperbolic or Elliptic Geometry. We shall not be inclined to quarrel with those who contend that this should be done, for we are quite in accord with the spirit of their ideals. But experience has shown that it is best, taking into account the mathematical immaturity of those for whom this book is intended, to avoid the confusion of what would have to be, if properly done, an excessively long, and possibly repellent, preliminary period of abstract reasoning. No attempt is made to conceal the deficiency. As a matter of fact it is carefully pointed out and the way left open for the student to remove it later.

The study of Non-Euclidean Geometry is a fine, rare experience. The majority of the students entering a class in this subject come, like the geometers of old, thoroughly imbued with what is almost a reverence for Euclidean Geometry. In it they feel that they have found, in all their studies, one thing about which there can be no doubt or controversy. They have never considered the logic of its application to the interpretation of physical space; they have not even surmised that it might be a matter of logic at all. What they are told is somewhat in the nature of a shock. But the startled discomposure of the first few days is rapidly replaced during the weeks which follow by renewed confidence, an eager enthusiasm for investigation, and a greater and more substantial respect for geometry for what it really is.

Nor is this all. Here some student may understand for the first time something of the nature, significance and indispensability of postulates, not only in geometry, but in the formation of any body of reasoned doctrine. He will recognize that not everything can be proved, that something must always be taken on faith, and that the character of the superstructure depends upon the nature of the postulates in the foundation. In *South Wind*, Norman Douglas has one of his characters say, “The older I get, the more I realize that everything depends upon what a man postulates. The rest is plain sailing.” Perhaps it is not too much to hope that a study of Non-Euclidean Geometry will now and then help some student to know this, and to formulate the postulates of his own philosophy consciously and wisely.

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