SAMPLE EXAM 1

Vectors are denoted by bold letters: $\mathbf{a}$, $\mathbf{v}$, $\mathbf{r}(t)$, etc.

1. Let $\mathbf{a} = \langle 1, 0, -1\rangle$, $\mathbf{b} = \langle -2, 3, 5\rangle$. Find $|\mathbf{a}|$, $\mathbf{a} - \mathbf{b}$, and a unit vector in the same direction as $\mathbf{a} - \mathbf{b}$.
2. Let $\mathbf{v} = \langle 5, -1, 6\rangle$, $\mathbf{w} = \langle -2, 2, -4\rangle$. Find $\mathbf{v} \cdot \mathbf{w}$ and the cosine of the angle between $\mathbf{v}$ and $\mathbf{w}$.
3. Which of the pairs of vectors $\{\mathbf{a}, \mathbf{b}\}$, $\{\mathbf{a}, \mathbf{c}\}$, $\{\mathbf{b}, \mathbf{c}\}$ are perpendicular? $\mathbf{a} = \langle 1, 2, 2\rangle$, $\mathbf{b} = \langle 8, -11, 7\rangle$, $\mathbf{c} = \langle -3, 1, 5\rangle$.
4. Suppose that $|\mathbf{v}| = 2$, $|\mathbf{w}| = 3$ and the cosine of the angle between $\mathbf{v}$ and $\mathbf{w}$ is $4/5$. Find $|\mathbf{v} \times \mathbf{w}|$.
5. Find the vector that is called the projection of $\mathbf{a}$ onto $\mathbf{b}$, using $\mathbf{a} = \langle 2, 2, 2\rangle$ and $\mathbf{b} = \langle 1, -1, -1\rangle$.
6. Compute the cross product of $\langle 3, 4, 5\rangle$ and $\langle -5, -1, 2\rangle$.
7. Find an equation of the plane containing the points $(1, 2, 1)$, $(-2, 0, -3)$, $(4, -1, 0)$.
8. Find an equation for the line that is the intersection of the planes $x + y + z = 2$ and $3x - 2y - z = -5$.
9. Find an equation for the plane that is perpendicular to both of the planes $x + y + z = 2$ and $3x - 2y - z = -5$ and contains the point $(1, 1, 1)$.
10. Let $\mathbf{r}(t) = \langle t^2 + 2, t^2 - 4t, 2t\rangle$. Find the tangent line at the point $(6, -4, 4)$.
11. Find the curvature of $\mathbf{r}(t)$ from the previous problem as a function of t and also find the curvature at $(6, -4, 4)$.
12. Suppose in $\mathbf{r}(t)$ from the previous two problems that t is time. Find the acceleration vector $\mathbf{a}(t)$. Find the scalar accelerations a_T and a_N.
13. Suppose an object moves so that its velocity vector is $\langle t, t^2, 1\rangle$, and at $t = 0$ it is at the point $(1, 1, 1)$. Where is it at $t = 1$?