Calculus III—Math 225

SAMPLE EXAM 2

1. Let $f(x, y) = \ln(x^2 + y^2)$. Compute the partial derivatives f_x, f_y, f_{xx}, f_{yy}, f_{xy}, f_{yx}.

2. Describe the level curves of $f(x, y) = \dfrac{x^2}{9} + \dfrac{y^2}{16}$.

3. Find an equation for the tangent plane to $z = e^y \ln x$ at $(1, 3, 0)$.

4. Use the "two-variable" version of the chain rule to compute $g'(t)$ if $g(t) = f(x, y)$, and $x = t^2$, $y = \cos t$.

5. Suppose $z = f(x, y)$. At a particular point (x_0, y_0), $\nabla f = \langle a, b\rangle$ is a vector. Describe the significance of both the length and the direction of this vector $\langle a, b\rangle$.

6. Find the directional derivative of $f(x, y, z) = x^2yz^3 + xy - z$ at the point $(1, 1, 1)$ in the direction indicated by the vector $\langle 1, 2, 3\rangle$.

7. Suppose $P(x, y, z) = x^4y - x^2y^3 + y\sin z$ gives the pressure at each point (x, y, z). At the point $(2, 1, \pi/4)$, in what direction does the pressure decrease most rapidly? Give your answer as a vector that points in the correct direction.

8. The point $(1, -2, 4)$ is on the surface described by $z = x^2y^2 + y\ln x$. Find a vector in one of the two possible directions to go from this point to stay on the level curve $z = 4$.

9. Find all critical points for $z = x\sin y$ and classify them as local maximum points, local minimum points, or saddle points.

10. Find the maximum and minimum values of $f(x, y) = x^2 - y^2$ above the curve given by $x^2 + 2y^2 = 1$.