Application 7:
Blind Signal Separation
(or Independent Component Analysis)

Setup: $X = SA$ (Tall, full rank)

Goal: Obtain S.
(Note: Solve $AX = B$ given only B...)

SOLUTION 1: $A = I$
Problem: Not too informative!

SOLUTION 2 (PCA):
Take $X^T X = \Psi \Lambda \Psi^T$. Then $S = X\Psi$, $A = \Psi^T$
Problem: S is uncorrelated, not independent...

Application 7, continued

Idea: Not only must $S^T S$ be diagonal (uncorrelated), but also $\Delta^k S^T \Delta^k S$ must be diagonal (implied by independence).

We assume that $\Delta^k S^T \Delta^k S = I$ for some k, but that $S^T S = \Lambda \neq cI$.

SOLUTION USING QSVD: Let $A = U\Sigma V^T$. Find U, Σ, V

$$\Delta X = (\Delta S)A = \Delta S U \Sigma V^T$$
$$\Delta X^T \Delta X = V\Sigma^2 V^T$$
$$\tilde{X} = XV\Sigma^{-1} = SU$$
$$\tilde{X}^T \tilde{X} = U^T S^T S U = U^T \Lambda U$$

Therefore, U, Σ, V are all computable from X!

Application 7, continued.

Connections to the QSVD: We don't have access to two matrices, only one... What if we use X and ΔX in the QSVD?

Then we have the factorizations:

$$X = U_X \Sigma_X P^T, \qquad \Delta X = V_X \Sigma_{dX} P^T$$

$$SA = U_X \Sigma_X P^T, \qquad \Delta SA = V_X \Sigma_{dX} P^T$$

<u>MAIN WORK:</u>

$$A = P^T$$

Clean (or "independent") Signal: U_X

Blind Signal Separation:

```
dX=diff(X);
[U,V,P,C,S]=gsvd(X,dX,0);  %Desired: U,P
```