Homework with 9.4

1. What is the difference between the "range", the "codomain", and the "image of the domain"?

2. Let $f(x) = 1 - 2x$. Show that f is $1-1$ and onto the real numbers, and show that it is continuous at $x = 1$.

3. Let $f(x) = 1 - x^2$.

   (a) Show (graphically and algebraically) that f is not 1-1.

   (b) Find the image of the interval $[-1, 2]$.

   (c) Find the preimage of the interval $[1/4, 3/4]$.

   (d) Find intervals A so that $f^2(A) = [1/4, 3/4]$.

   (e) Let $A = [0, 2]$ and $B = [-1, 1]$. Show graphically what the following equation means (and algebraically, what is it?)

$$f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$$

4. Show, by means of an example, that if f is not continuous, then the image of a closed interval could be either closed or open (an open interval is one that does not contain its endpoints) or neither.

5. As in our text, define

$$I_{s_0 s_1} = \{x \in I \,|\, x \in I_{s_0}, Q_c(x) \in I_{s_1}\}$$

   Show graphically (along the line $y = x$) where I_{01} is, and show that

$$I_{01} = I_0 \cap Q_c^{-1}(I_1)$$

6. Continuing the last problem, show that:

$$I_{010} = I_0 \cap Q_c^{-1}(I_{10})$$

7. (Same notation as last two problems) Show that

$$I_{010} \subset I_{01} \subset I_0$$

8. Let g be defined piecewise as:

$$g(x) = \begin{cases} x & \text{if } 0 \le x \le 1 \\ x - 1 & \text{if } 2 < x \le 3 \end{cases}$$

   Define $g : [0, 1] \cup (2, 3] \to [0, 2]$.

   (a) Show that g is $1-1$ and onto (which makes g invertible).

   (b) Find g^{-1} and conclude that g is continuous, but g^{-1} is not.