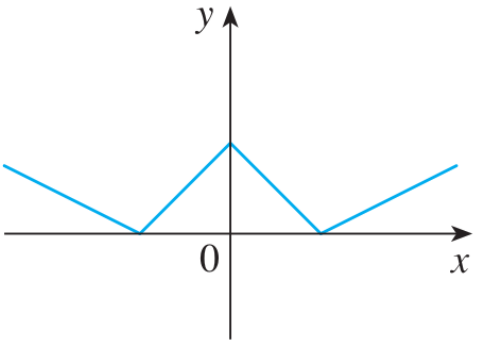
Math 125 Exam 2, Sample 3

Show all your work! Answers with no work may get no credit. No calculators or notes allowed.

1. Given the graph of $f(x)$ below, sketch a graph of $f'(x)$.



2. Differentiate:
   (a) $y = \log_2(x)\sec(x)$
   (b) $y = \sqrt{\sin(\sqrt{x})}$
   (c) $y = x3^{-1/x}$
   (d) $y = \frac{2x+5}{5^x}$

3. Derive the formula for the derivative of $\cos^{-1}(x)$. We started by: $y = \cos^{-1}(x)$, which is $\cos(y) = x$. We can then differentiate implicitly- Fill in the rest of the argument.

4. Find the equation of the tangent line to $\sqrt{y} + xy^2 = 5$ at the point $(4, 1)$.

5. Differentiate: $y = x^{\cos(x)}$

6. True or False, and explain:
   (a) The derivative of a polynomial is a polynomial.
   (b) If f is differentiable, then $\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$
   (c) The derivative of $y = \sec^{-1}(x)$ is the derivative of $y = \cos(x)$.
   (d) $\frac{d}{dx}(10^x) = x10^{x-1}$
   (e) If $y = \ln|x|$, then $y' = \frac{1}{x}$
   (f) The equation of the tangent line to $y = x^2$ at $(1, 1)$ is: $y - 1 = 2x(x - 1)$
   (g) If $y = e^2$, then $y' = 2e$
   (h) If $y = ax + b$, then $\frac{dy}{da} = x$

7. If $f(x) = 2x + \mathrm{e}^x$, find the equation of the tangent line **to the inverse** of f at $(1, 0)$. HINT: Do not try to compute f^{-1} algebraically.

8. Radium-226 has a half-life of 1600 years. How long does it take for 18 grams of Radium-226 to decay to leave a total of 2.25 grams? (You can use a calculator for this practice problem- the numbers will work out nicely on the exam).

9. If a snowball melts so that its surface area decreases at a rate of 1 cm^2/min, find the rate at which the diameter decreases when the diameter is 10 cm. The surface area of a sphere is given by $SA = 4\pi r^2$, where r is the radius.