Summary for Exam 1 (Up through 2.8)

Ch 1, App D: Precalc topics

1. Construct the equation of a line (pt-slope form), Use the quadratic formula, find trigonometric values from right triangles/unit circle, function composition, simplify exponentials/logs using rules of exponents/logs. Be able to compute "nice" inverse trig function values.

2. Be able to factor a quadratic. Be able to algebraically solve an equation.

3. Definitions: Piecewise defined functions, $|x|$, "one-to-one", "natural domain"

4. Be able to "Find the domain".

5. Use a **sign chart** to determine where some product of factors is positive/negative.

6. Know the difference between "inverse of a function" and the reciprocal of a function. Be able to compute an inverse graphically and algebraically.

7. Function notation: Given a formula for $f(x)$, be able to compute expressions like $f(2+h)$.

The Limit

1. **Key definition: The Limit**

   $\lim_{x\to a} f(x) = L$ means that we can keep the $f(x)$ values arbitrarily close to L by keeping the $x-$values sufficiently close to a.

2. Be able to compute limits algebraically and graphically. Be able to compute right and left-hand limits.

3. Algebraic Methods to compute limits:

   (a) Simplify (e.g., absolute values)
   (b) Factor and Cancel
   (c) Multiply by Conjugate
   (d) Divide by x^n (Mainly for $x \to \infty$). Be careful! $x = \sqrt{x^2}$ if $x \geq 0$, but if $x < 0$, $x = -\sqrt{x^2}$

4. The Squeeze Theorem.

5. Horizontal/Vertical Asymptotes:

   (a) $x = a$ is a vertical asymptote for $f(x)$ if one of the following limits is infinite: $\lim_{x\to a^{\pm}} f(x)$
   (b) $y = b$ is a horizontal asymptote for $f(x)$ if one of the following is true: $\lim_{x\to\pm\infty} f(x) = b$

   Our template function: $\lim_{x\to\pm\infty} \frac{1}{x^r} = 0, \quad r > 0$

   (Note: x^r needs to be computable if $x \to -\infty$) Also, in a similar vein: $\lim_{x\to\infty} \mathrm{e}^{-x} = 0$ The inverse tangent has horizontal asymptotes:

   $$\lim_{x\to\pm\infty} \tan^{-1}(x) = \pm\frac{\pi}{2}$$

   And in general, if a function has a vertical asymptote at $x = a$, its inverse function will have a horizontal asymptote at $y = a$.

6. Intuition that can be used:

(a) "$\infty + \infty = \infty$", but $\infty - \infty$ is not necessarily 0. (Similarly, the product but not the quotient)
(b) If the denominator goes to zero, but the numerator does not, the limit is $\pm\infty$.
(c) If the denominator goes to $\pm\infty$, and the numerator does not, the overall limit goes to zero.
(d) Given a rational function, if the degree of the numerator is larger than the denominator, the function goes to $\pm\infty$ as $x \to \pm\infty$.
If the degree of the denominator is larger, then the function goes to zero (again, as $x \to \pm\infty$)

7. Limit Laws (Sect 1.3): Understand them well enough so that you can compute a limit algebraically (I won't ask you to list them, only work with them).

Continuity

1. **Key Definition: Continuity**
   f is continuous at $x = a$ if $\lim_{x \to a} f(x) = f(a)$
2. Interpretation of the definition: This means 3 things: (1) $f(a)$ exists, (2) The limit exists, and (3) Items 1 and 2 are the same number.
3. Show that a function is not continuous at a point by stating which of the three parts are violated. State if a function is "continuous from the right" or left.
4. Theory about continuous functions: Know that our usual functions (see the list in Theorem 7) are all continuous *on their domain*. Know that the sum/difference, product/quotient of continuous functions is continuous (with a possibly restricted domain)
5. Be able to state and use the **Intermediate Value Theorem**:
   If f is continuous on $[a, b]$, and N is a number between $f(a)$ and $f(b)$, there is at least one c in $[a, b]$ so that $f(c) = N$.
   In practice, we usually use the IVT as:
   If f is continuous, and $f(x_1) > 0, f(x_2) < 0$, then there is a c between x_1 and x_2 where $f(c) = 0$ (f has at least one root in the interval between x_1 and x_2).

The Derivative

1. If $f(x)$ is a generic function, its average rate of change on the interval $[a, b]$ is: $\frac{f(b)-f(a)}{b-a}$. Be able to write average velocity if $f(t)$ is displacement.
2. **Key Definition: The Derivative**
   The instantaneous rate of change of f (or instantaneous velocity of $f(t)$) is the **derivative** of f at point a, defined by:
   $$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$
   Be able to compute this given numerical values of a, or as an arbitrary value of a (you would be given $f(x)$).
3. Interpretations of the Derivative of f at $x = a$:
   (a) The velocity at $x = a$.
   (b) The slope of the tangent line at $(a, f(a))$.
   (c) The instantaneous rate of change of f at $x = a$.
4. Equation of the Tangent Line at $x = a$: This is the line going through $(a, f(a))$ with slope $f'(a)$. The best (and fastest) way to write the line: $y - f(a) = f'(a)(x - a)$