M125
Fall 2003

Quiz 10

Name:__________

Show all your work!

1. Determine the intervals where f is concave up, and where it is concave down: $f(x) = 3x^5 - 5x^3 + 3$

   First, find the second derivative. Then use a sign chart to determine where it is positive and where it is negative.

$$f'(x) = 15x^4 - 15x^2 \quad \Rightarrow \quad f''(x) = 60x^3 - 30x$$

   Factoring f'' for the sign chart:

$$f''(x) = 30x(2x^2 - 1) = 30x(\sqrt{2}x - 1)(\sqrt{2}x + 1)$$

   The sign chart:

$30x$	$-$	$-$	$+$	$+$
$(\sqrt{2}x - 1)$	$-$	$-$	$-$	$+$
$(\sqrt{2}x + 1)$	$-$	$+$	$+$	$+$
	$x \le -\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}} \le x \le 0$	$0 \le x \le \frac{1}{\sqrt{2}}$	$x > \frac{1}{\sqrt{2}}$

   Overall, f is concave up if $-\frac{1}{\sqrt{2}} \le x \le 0$ or if $x > \frac{1}{\sqrt{2}}$. The function f is concave down on the other intervals, $x \le -\frac{1}{\sqrt{2}}$ or $0 \le x \le \frac{1}{\sqrt{2}}$

2. For the given function on the given interval, find the point c guaranteed by the Mean Value Theorem:

$$f(x) = \frac{x}{x+2}, \qquad [1, 4]$$

   The Mean Value Theorem states, in this case, that there is a c so that:

$$\frac{f(4) - f(1)}{4 - 1} = f'(c)$$

   Computing these, the left hand side is $\frac{1}{3}\left(\frac{2}{3} - \frac{1}{3}\right) = \frac{1}{9}$. The derivative is:

$$f'(c) = \frac{2}{(x+2)^2}$$

   Equating these,

$$\frac{2}{(x+2)^2} = \frac{1}{9} \qquad 18 = (x+2)^2 \qquad x = -2 + \sqrt{18}$$

   (the other solution for x is not in the interval)