M125
Fall 2003

Quiz 8

SOLUTION

Show all your work!

1. A 300 gallon tank contains 100 gallons of brine with a concentration of 1 pound per gallon of water. A brine containing $\frac{1}{2}$ pounds of salt per gallon of water runs into the tank at a rate of four gallons per minute. The well-stirred mixture runs out of the tank at the same rate. When is the concentration of salt in the tank $\frac{3}{4}$ pounds per gallon?

SOLUTION: Let $Q(t)$ be the pounds of salt in the tank at time t. Given the first sentence, there is initially 100 pounds of salt, so we know that $Q(0) = 100$.

Now, the rate of change is given by:

$$Q' = \frac{4 \text{ gal}}{1 \text{ min}} \cdot \frac{\frac{1}{2} \text{ pound}}{1 \text{ gal}} - \frac{4 \text{ gal}}{1 \text{ min}} \cdot \frac{Q \text{ pounds}}{100\text{gal}} = 2 - \frac{1}{25}Q$$

Therefore,

$$(Q-50)' = -\frac{1}{25}(Q-50)$$

so that

$$Q(t) = A\mathrm{e}^{-\frac{1}{25}t} + 50$$

Use the initial condition to solve for A,

$$100 = A + 50$$

so that $A = 50$. Now, we have the complete solution to the differential equation,

$$Q(t) = 50\mathrm{e}^{-\frac{1}{25}t} + 50$$

When is the concentration $\frac{3}{4}$? This would mean that there is: $\frac{3}{4} \cdot 100 = 75$ pounds of salt:

$$75 = 50\mathrm{e}^{-\frac{1}{25}t} + 50 \quad \Rightarrow \quad \frac{1}{2} = \mathrm{e}^{-\frac{1}{25}t}$$

so that:

$$-\ln(2) = -\frac{1}{25}t \quad \Rightarrow \quad t = 25\ln(2) \approx 17.33$$

It takes approximate 17.33 minutes for the tank to reach the desired concentration.