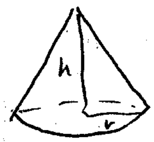
Examples, Section 3.9

1. Sand falls from an overhead bin, accumulating in a conical pile with a radius that is always three times the height. If the sand falls from the bin at a rate of 120 cubic feet per minute, how fast is the height changing when the pile is 10 feet tall? (Hint: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$)

SOLN:



$r = 3h$, so

$V = \frac{1}{3}\pi (3h)^2 h = 3\pi^3$

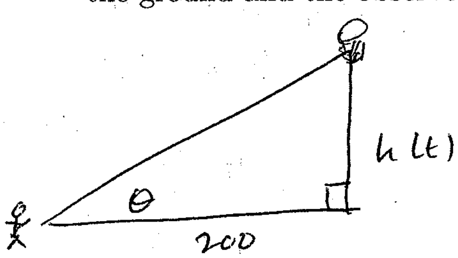
We want to find $\frac{dh}{dt}$ when $h = 10$, $dV/dt = 120$:

$$\frac{dV}{dt} = 9\pi h^2 \frac{dh}{dt}$$

$$120 = 9\pi (10)^2 \frac{dh}{dt}$$

So $\frac{dh}{dt} \approx 0.042$ ft/min.

2. An observer stands 200 meters from the launch site of a hot air balloon. The balloon rises vertically at a constant rate of 4 m/s. How fast is the angle of elevation of the balloon increasing 30 seconds after launch (the angle of elevation is the angle between the ground and the observer's line of sight to the balloon).



$$\tan\theta = \frac{h}{200}$$

So Find $\frac{d\theta}{dt}$ 30 sec after launch.

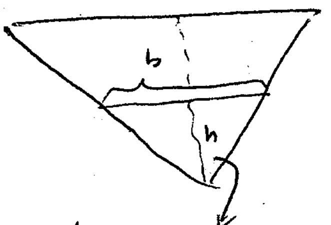
Taking deriv's, $\sec^2\theta \frac{d\theta}{dt} = \frac{1}{200}\frac{dh}{dt}$, so we need $\sec\theta$. Since $\frac{dh}{dt} = 4$, after 30 secs, $h = 4 \cdot 30 = 120$.

So the hypotenuse is $\sqrt{200^2 + 120^2} \approx 233.24$.

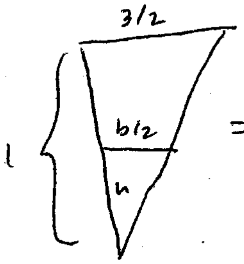
So $\cos\theta \approx 0.86$, ~~and~~

$$\frac{d\theta}{dt} = \frac{4(0.86)^2}{200}$$

3. (Ex 24) A trough is 10 ~~meters~~ feet long and its cross section has the shape of an isosceles triangle that is 3 feet across at the top and has a height of 1 foot. If the trough is being filled at a rate of 12 ft^3/min, how fast is the water level rising when the water is 6 inches deep?



$V = \frac{1}{2} b h \cdot 10 = 5 b h$



$\Rightarrow \frac{\frac{3}{2}}{1} = \frac{b/2}{h} \Rightarrow b = 3h$

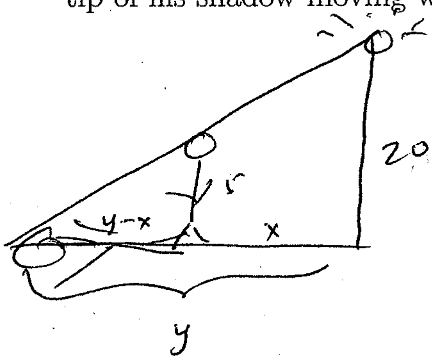
$V = 5(3h)h = 15h^2$

So $\frac{dV}{dt} = 30h \frac{dh}{dt}$

$12 = 30(1/2) \frac{dh}{dt}$

$\frac{dh}{dt} = 4/5$

4. A street light is mounted at the top of a 20 foot pole. A man 5 feet tall walks away from the pole with a speed of 5 feet per second along a straight path. How fast is the tip of his shadow moving when he is 30 feet from the pole?



$\Rightarrow \frac{20}{y} = \frac{5}{y-x} \Rightarrow 20y - 20x = 5y$

$15y = 20x$

$3y = 4x$

$y = 4/3\, x$

$\frac{dy}{dt} = \frac{4}{3} \frac{dx}{dt}$

$\Rightarrow \frac{dy}{dt} = \frac{4}{3} \cdot 5 = \frac{20}{3}$