M126
Fall 2025

Quiz 4

Name: key

Show all your work! No calculators or other notes allowed.. Here are the integral formulas from section 1.7:

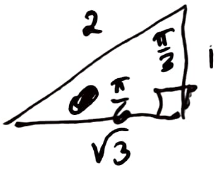
$$\int \frac{du}{\sqrt{a^2-u^2}} = \sin^{-1}\left(\frac{u}{a}\right)+C \qquad \int \frac{du}{a^2+u^2} = \frac{1}{a}\tan^{-1}\left(\frac{u}{a}\right)+C \qquad \int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{|u|}{a}\right)+C$$

1. Evaluate each integral:

(a) $\displaystyle\int \frac{dx}{25+16x^2}$ $\Rightarrow$ $a = 5$, $u = 4x$ $\Rightarrow$ $\frac{1}{5}\tan^{-1}\left(\frac{4x}{5}\right)+C$

(b) $\displaystyle\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{1}{1+x^2}\,dx = \tan^{-1}(x)\Big|_{1/\sqrt{3}}^{\sqrt{3}} = \tan^{-1}(\sqrt{3}) - \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$= \frac{\pi}{3} - \frac{\pi}{6} = \boxed{\frac{\pi}{6}}$



(c) $\displaystyle\int \frac{\tan^{-1}(2t)}{1+4t^2}$ $\Rightarrow$ Let $u = \tan^{-1}(2t)$,

$du = \frac{1}{1+(2t)^2}\cdot 2\,dt$

$\frac{1}{2}\int u\,du = \frac{1}{4}u^2 + C$

$= \boxed{\frac{1}{4}\left(\tan^{-1}(2t)\right)^2 + C}$

2. (2 pts extra credit) What's wrong with the integral: $\displaystyle\int_1^2 \frac{dt}{\sqrt{1-t^2}}$?

$\frac{1}{\sqrt{1-t^2}}$ is not continuous on $[1,2]$ (so FTC can't be applied)