

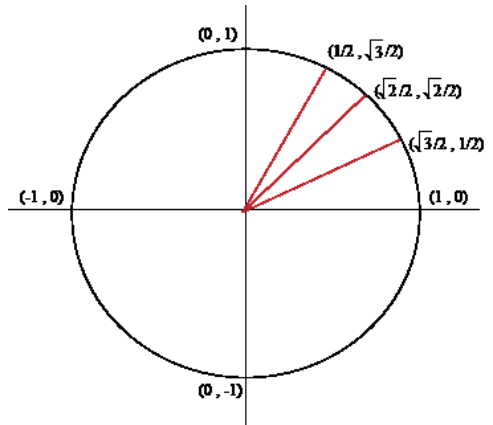
Formulas

$$\sum_{i=1}^n i = \frac{n(n+1)}{2} \quad \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$L_n = \sum_{i=1}^n f\left(a + (i-1)\frac{b-a}{n}\right) \Delta x$$

$$R_n = \sum_{i=1}^n f\left(a + i\frac{b-a}{n}\right) \Delta x$$

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x$$



$$\frac{d}{dx} \int_a^x f(t) dt = f(x) \quad \int_a^b F'(x) dx = F(x)|_a^b = F(b) - F(a) \quad f_{avg} = f(c) = \frac{1}{b-a} \int_a^b f(x) dx$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$\tan^2(x) + 1 = \sec^2(x)$$

$$\sin(2x) = 2 \sin(x) \cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$

$$\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$

Given:	Guess:
$a^2 - u^2$	$u = a \cdot \sin(\theta)$
$u^2 + a^2$	$u = a \cdot \tan(\theta)$
$u^2 - a^2$	$u = a \cdot \sec(\theta)$

$$L = \int_a^b \sqrt{1 + (f'(x))^2} dx$$

$$SA = \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2} dx$$

$$\bullet \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\bullet \int \frac{dx}{ax + b} = \frac{1}{a} \ln |ax + b| + C$$

$$\bullet \int \tan(u) du = \ln |\sec(u)| + C$$

$$\bullet \int \sec(u) du = \ln |\sec(u) + \tan(u)| + C$$

$$\bullet \int \cot(u) du = \ln |\sin(u)| + C$$

$$\bullet \int \csc(u) du = \ln |\csc(u) - \cot(u)| + C$$

$$\bullet \int \tan^n(u) du = \frac{1}{n-1} \tan^{n-1}(u) - \int \tan^{n-2}(u) du$$

$$\bullet \int \sec^n(u) du = \frac{1}{n-1} \sec^{n-2}(u) \tan(u) + \frac{n-2}{n-1} \int \sec^{n-2}(u) du$$

$$\text{Disk: } \int \pi r^2 \cdot \text{thickness}$$

$$\text{Washer: } \int \pi(R^2 - r^2) \cdot \text{thickness}$$

$$\text{Shells: } \int 2\pi r h \cdot \text{thickness}$$

Hint sheet for Sequences and Series

This hint sheet is meant to remind you about the convergence/divergence tests, and is not a step by step guide. This sheet will be available to you during the exam.

1. Known series:

- A geometric series converges when $|r| < 1$, and diverges if $|r| \geq 1$.
- The p -series converges if $p > 1$, and diverges if $p \leq 1$.
- The similarity of an unknown series to either a geometric series or a p -series is useful for the tests for convergence.

2. Convergence/Divergence tests:

(a) Test for Divergence.

(b) Special structure: Telescopic (not common, but is sometimes convenient).

(c) Tests for positive series (or absolute convergence):

i. The Integral Test.

Use for positive decreasing terms that come from a function you can integrate.

ii. The Comparison Tests:

- Direct Comparison Test.

Compare to a known positive series (geo or p -series, for example). Works best when one term is clearly larger/smaller.

- Limit Comparison Test:

Use when direct comparison is awkward but a_n behaves like a simpler b_n .

iii. Root Test

iv. Ratio Test

NOTE: The results of the Ratio and Root tests are the same (in terms of the limit), and are similar to the results of the Geo series “test”.

(d) Alternating Series Test (for conditional convergence)

For terms with $(-1)^n$ or $(-1)^{n+1}$. Check that the positive part decreases to 0.

Given a function $f(x)$ at a point $x = a$, the Taylor series and the remainder (using $T_n(x)$) are given below. In the remainder, c is some number between x and a .

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!} (x - a)^n \qquad R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!} (x - a)^{n+1}$$

Template series were for: $\frac{1}{1-x}$, e^x , $\sin(x)$ and $\cos(x)$.