

**Final Exam Review**  
**Calculus II**  
**Sheet 1**

1. Prove by induction:  $\sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}$
2. State the definition of  $\int_a^b f(x) dx$ .
3. True or False, and give a short reason:
  - (a) The Alternating Series Test is sufficient to show that a series is conditionally convergent.
  - (b) You can use the Integral Test to show that a series is absolutely convergent.
  - (c) Consider  $\sum a_n$ . If  $\lim_{n \rightarrow \infty} a_n = 0$ , then the sum is said to converge.
  - (d) The sequence  $a_n = 0.1^n$  converges to  $\frac{1}{1-0.1}$
4. Set up an integral for the volume of the solid obtained by rotating the region defined by  $y = \sqrt{x-1}$ ,  $y = 0$  and  $x = 5$  about the  $y$ -axis. Find the work involved if this was a tank filled with water and we wanted to pump it all out of the top (You may assume the measurements are in meters, and  $g = 9.8$ . The density of water is  $1000 \text{ kg/m}^3$ ).
5. Write the area under  $y = \sqrt[3]{1+x}$ ,  $1 \leq x \leq 4$  as the limit of a Riemann sum (use **right** endpoints). For the same function, write an integral representing the arc length (do not evaluate the integral).
6. Find the Taylor series for  $f(x) = \sqrt{x}$  centered at  $a = 9$ .
7. Find  $\frac{dy}{dx}$ , if  $y = \int_{\cos(x)}^{5x} \cos(t^2) dt$
8. Let  $f(x) = e^x$  on the interval  $[0, 2]$ . (a) Find the average value of  $f$ . (b) Find  $c$  such that  $f_{\text{avg}} = f(c)$ .
9. Use a template series to find the series for  $\int \cos(x^2) dx$ .
10. Does the series converge (absolute or conditional), or diverge?
  - (a)  $\sum_{n=1}^{\infty} \frac{\cos(n/2)}{n^2 + 4n}$
  - (b)  $\sum_{n=1}^{\infty} \frac{n^2 + 1}{5^n}$
  - (c)  $\sum_{n=1}^{\infty} \frac{3^n n^2}{n!}$
11. Find the interval of convergence:
  - (a)  $\sum_{n=1}^{\infty} \frac{n^2 x^n}{10^n}$
  - (b)  $\sum_{n=1}^{\infty} \frac{(3x-2)^n}{n3^n}$
  - (c)  $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n-1}}{(2n-1)!}$
12. Evaluate the integral. You should be able to do these without the table of integrals.

$$\begin{array}{lll} \text{(a)} \int \frac{1}{y^2 - 4y - 12} dy & \text{(c)} \int x^2 \cos(3x) dx & \text{(e)} \int \frac{dx}{x \ln(x)} \\ \text{(b)} \int \frac{2}{3x+1} + \frac{2x+3}{x^2+9} dx & \text{(d)} \int_{-2}^2 |x-1| dx & \text{(f)} \int x\sqrt{x-1} dx \end{array}$$

13. The velocity function is  $v(t) = 3t - 5$ ,  $0 \leq t \leq 3$  (a) Find the displacement. (b) Find the distance traveled.