Review: Exam 3

1. Goals for this portion of the course:
   - Be able to compute the area between curves, the volume of solids of revolution, and understand the mean value of a function. We had three basic volumes: Disk, Washer and Shell.
   - Stimulate mental connections between geometry and calculus- Given a picture, write down the formula for the given area or volume of a solid. Apply algebra if needed to obtain the required information (i.e., points of intersection). Also, be able to go from the integral of an area/volume to a geometric picture.
   - Be able to finish the integration to obtain a final solution.
   - Understand and apply u, du substitution to find antiderivatives (and evaluate the integral).
2. Material to be covered: 6.1, 6.2, 6.3, 6.5 plus review of u, du substitution.

Review Questions:

1. Find the area of the region bounded between the curves:
   (a) $y = \mathrm{e}^x - 1$, $y = x^2 - x$, $x = 1$
   (b) $x - 2y + 7 = 0$, $y^2 - 6y - x = 0$
2. Find the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis. You may use any method.
   (a) $y = \sqrt{x-1}$, $y = 0$, $x = 3$, about the x-axis.
   (b) $y = \mathrm{e}^{-2x}$, $y = 1 + x$, $x = 1$, about the x-axis.
   (c) $y = x^3$, $y = 8$, $x = 0$, about the $y-$axis.
3. If f is continuous, and $\int_1^3 f(x)\,dx = 8$, so that f takes on the value 4 at least once on the interval $[1, 3]$.
4. Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the given curves about the specified axis. You may use any method.
   (a) $y = x^3$, $y = x^2$, about $y = 1$
   (b) $y = x^3$, $y = 8$, $x = 0$ about $x = 2$.
5. Each integral represents the volume of a solid. Describe the solid (with words and/or pictures).
   (a) $\int_0^{\pi/2} 2\pi x \cos(x)\,dx$

(b) $\displaystyle\int_0^{\pi/2} 2\pi \cos^2(x)\, dx$

(c) $\displaystyle\int_0^1 \pi \left[(2-x^2)^2 - (2-\sqrt{x})^2\right]\, dy$

6. Find the average value of $f(x) = x^2\sqrt{1+x^2}$ on the interval $[0, 2]$.

7. Let R be the region in the first quadrant bounded by the curves $y = x^3$ and $y = 2x - x^2$. Calculate the following quantities:

   (a) The area of R.

   (b) The volume obtained by revolving R about the $x-$axis.

   (c) The volume obtained by revolving R about the $y-$axis.

8. Find the numbers b so that the average value of $f(x) = 2 + 6x - 3x^2$ on the interval $[0, b]$ is equal to 3.

9. Let R_1 be the region bounded by $y = x^2$, $y = 0$, and $x = b$, with $b > 0$. Let R_2 be the region bounded by $y = x^2$, $x = 0$, and $y = b^2$ (Same b).

   (a) Is there a b so that R_1 and R_2 have the same area?

   (b) Is there a value of b so that R_1 sweeps out the same volume if R_1 is rotated about the $x-$axis versus rotating R_1 about the $y-$axis?

   (c) Is there a value of b so that R_1 and R_2 sweep out the same volume when rotated about the $y-$axis?

10. Evaluate:

    (a) $\displaystyle\int x\sqrt{x-1}\, dx$

    (b) $\displaystyle\int \frac{x+1}{x^2+2x}\, dx$

    (c) $\displaystyle\int \frac{x}{\sqrt[4]{x+2}}\, dx$

    (d) $\displaystyle\int \frac{x^2}{\sqrt{1-x}}\, dx$