Integration By Parts: Using a Table

Often, we need to do integration by parts several times to obtain an antiderivative. This shows you how to do it using a table, and you will find it very convenient. Note, however, there are times when a table shouldn't be used, and we'll see examples of that as well.

The general idea: Integration by parts uses the formula:

$$\int u\,dv = uv - \int v\,du$$

Notice that this can be put into a table:

sign	Differentiate	Antidifferentiate
+	u	dv
-	du	v

where we get the same formula by going diagonally from u, multiply by v, and then take the minus sign and integrate the product going straight across.

We'll show how to use the chart by the use of several examples:

Example 1: $\int x^2 e^{-2x}\,dx$

We'll use the chart, and differentiate on x^2:

$+$	x^2	e^{-2x}
$-$	$2x$	$-\frac{1}{2}e^{-2x}$
$+$	2	$\frac{1}{4}e^{-2x}$
$-$	0	$-\frac{1}{8}e^{-2x}$

$$\Rightarrow -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + \frac{1}{8}\int 0 \cdot e^{-2x}\,dx$$

Example 2: $\int x\sin(2x)\,dx$

$+$	x	$\sin(2x)$
$-$	1	$-\frac{1}{2}\cos(2x)$
$+$	0	$-\frac{1}{4}\sin(2x)$

$$\Rightarrow -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) - \frac{1}{4}\int 0 \cdot \sin(2x)\,dx$$

Example 3: $\int e^{2x}\sin(3x)\,dx$

This is a special case. We know we'll have to integrate by parts twice, then look at what we get:

$+$	e^{2x}	$\sin(3x)$
$-$	$2e^{2x}$	$-\frac{1}{3}\cos(3x)$
$+$	$4e^{2x}$	$-\frac{1}{9}\sin(3x)$

$$\Rightarrow -\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{9}e^{2x}\sin(3x) - \frac{4}{9}\int e^{2x}\sin(3x)\,dx$$

Rewriting this, we see that:

$$\int e^{2x}\sin(3x)\,dx = -\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{9}e^{2x}\sin(3x) - \frac{4}{9}\int e^{2x}\sin(3x)\,dx$$

so that:

$$\frac{13}{9}\int e^{2x}\sin(3x)\,dx = -\frac{1}{3}e^{2x}\cos(3x) + \frac{2}{9}e^{2x}\sin(3x)$$

And we arrive at the final answer:

$$\int e^{2x}\sin(3x)\,dx = -\frac{3}{13}e^{2x}\cos(3x) + \frac{2}{13}e^{2x}\sin(3x)$$