Final Exam Review
Calculus II
Sheet 2

1. Short Answer:
   (a) Compare and contrast the "Limit Comparison Test" to the "Ratio Test".
   (b) Suppose the power series $\sum c_n x^n$ converges at $x = 3$. What are all the other values for which we know the series must converge?
   (c) If $\sum a_n$, $\sum b_n$ are series with positive terms, and a_n, b_n both go to zero as $n \to \infty$, then what can we conclude if $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$?
   (d) What is the derivative of $\sin^{-1}(x)$? Of $\tan^{-1}(x)$? What is the antiderivative of each?
   (e) What was the Mean Value Theorem for Integrals?
2. Suppose $h(1) = -2$, $h'(1) = 2$, $h''(1) = 3$, $h(2) = 6$, $h'(2) = 5$, and $h''(2) = 13$, and h'' is continuous. Evaluate $\int_1^2 h''(u)\, du$.
3. Let R be the region in the first quadrant bounded by $y = x^3$ and $y = 2x - x^2$. Calculate: (a) The area of R, (b) Volume obtained by rotating R about the $x-$axis (c) Volume obtained by rotating R about the $y-$axis
4. Find the volume of the solid obtained by rotating the region bounded by: $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 3$ about $y = -1$.
5. Write the area under $y = \sqrt[3]{x}$, $0 \leq x \leq 8$ as the limit of a Riemann sum (use right endpoints).
6. Compute $\frac{dg}{dy}$, if $g(y) = \int_3^{\sqrt{y}} \frac{\cos(t)}{t}\, dt$.
7. Compute the limit, by using the series for $\sin(x)$: $\lim_{x\to 0} \frac{\sin(x)}{x}$
8. Use the appropriate series to integrate: $\int \mathrm{e}^{x^2}\, dx$

Does the given series converge (abs or cond) or diverge?

9. $\sum_{n=1}^{\infty} \left(\frac{3n}{1+8n}\right)^n$
10. $\sum_{k=1}^{\infty} (-1)^k \frac{\sqrt{k}}{k+5}$
11. $\sum_{n=1}^{\infty} \frac{\sqrt{n}+\sqrt[3]{n}}{n^2+n^3}$

Evaluate, or state that it diverges.

12. $\int \mathrm{e}^{-x} \sin(2x)\, dx$
13. $\int \ln(x)\, dx$
14. $\int_0^3 \frac{1}{\sqrt{x}}\, dx$
15. $\int \sin^2 x \cos^5 x\, dx$

Find the interval of convergence:

16. $\sum_{n=2}^{\infty} \frac{x^n}{n(\ln(n))^2}$
17. $\sum_{n=1}^{\infty} \frac{(-1)^n x^{2n}}{(n!)^2 2^{2n}}$
18. Find the radius of convergence: $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n!)} x^n$