Review Questions, Calc I and App. E, 5.1-5.2

Here are the items from Calc I you should be familiar with:

Topic:	Section
Definition of continuity	Section 2.5, p. 122
Where is $f(x)$ continuous?	Theorem 7, p. 127
What is the domain of $f(x)$?	Section 1.1, p. 12 Appendix A
$\lim_{x\to\infty} f(x)$ (horizontal asympt)	Section 2.6, p. 137 Examples 3, 4,5
Some antiderivatives	Section 4.10

Test your understanding by answering the following questions from this material!

1. If $f(x) = \sqrt{\frac{1-x^2}{x^2-4}}$, where is f continuous?
2. Compute the limit, if it exists:
   (a) $\lim_{n\to\infty} \dfrac{3n^3+5n^2+2n}{6n^3+2n+1}$
   (b) $\lim_{n\to\infty} \dfrac{1-\sqrt{n}}{1+\sqrt{n}}$
   (c) $\lim_{n\to\infty} 6 + \dfrac{18}{5n^2}\cdot n(n+2)$
   (d) $\lim_{x\to\infty} \sqrt{x^2+1} - \sqrt{x^2-1}$ (This one is included for extra practice only)
3. The following questions give a Riemann Sum. What definite integral does each represent?
   (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{1}{n}\left[\left(\dfrac{i}{n}\right)^3 + 1\right]$
   (b) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{3}{n}\sqrt{1+\dfrac{3i}{n}}$
   (c) $\lim_{n\to\infty} \dfrac{1}{n}\sum_{i=1}^{n} \dfrac{1}{1+(i/n)^2}$
4. Do the last problem again, but now make your interval begin with $a = -1$.
5. Do problem 3 again, but now make every interval $a = 2$, $b = 6$
6. Give the general antiderivative, $F(x)$, where $f(x)$ is given as:
   (a) $f(x) = 2x + 5(1-x^2)^{-1/2}$
   (b) $f(x) = \frac{3}{x^2} - \frac{5}{x^4} + \sqrt[3]{x^2}$
   (c) $f(x) = 4 - 3(1+x^2)^{-1}, F(1) = 0$ (Give the specific antiderivative)
7. Set up, but do not evaluate, the integral of the given function on the given interval *using the definition*:
   (a) $f(x) = 1 + x, 2 \le x \le 3$
   (b) $f(x) = \cos(x), \pi \le x \le 2\pi$
   (c) $f(x) = 2 + x + 3x^2, -1 \le x \le 3$
8. Evaluate each limit. You may use the formulas:

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

   (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{1}{n}\left(\dfrac{i}{n}\right)^2$
   (b) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{1}{n}\left(\dfrac{i^3}{n^3}+1\right)$
   (c) $\lim_{n\to\infty} \sum_{i=1}^{n} \dfrac{2}{n}\left[\left(\dfrac{2i}{n}\right)^3 + 5\left(\dfrac{2i}{n}\right)\right]$
9. Check your answers to the previous problem by first setting up, then evaluating, the definite integral that each limit represents.