Homework Hints, Section 7.8

5. Let $u = x - 2$.

7. Let $u = 3 - 4x$

10. Recall that $\int 2^r \, dr = 2^r / \ln(r)$. (Converges)

14. Let $u = \sqrt{x}$. (Converges)

20. Integrate by parts. Use l'Hospital's rule for the limit. (Converges)

25. $u = \ln(x)$.

29. Either integrate directly or use $u = x + 2$

31. Break it up- There's a vertical asymptote at $x = 0$.

41. Area is $\int_1^\infty e^{-x} \, dx$

49. Note that $\dfrac{x}{x^3+1} < \dfrac{x}{x^3}$

50. Note that $\frac{2+e^{-x}}{x} > \frac{2}{x}$ (Diverges).

55. Break it up and deal with the integrals separately

$$\int_0^1 \frac{1}{\sqrt{x}(1+x)} \, dx + \int_1^\infty \frac{1}{\sqrt{x}(1+x)} \, dx$$

To integrate, let $u = \sqrt{x}$, so $u^2 = x$, etc. You should get $2\tan^{-1}(u)$ for the integral.

56. Note that there is a discontinuity at $x = 2$, so break it up as

$$\int_2^3 \frac{dx}{x\sqrt{x^2-4}} + \int_3^\infty \frac{dx}{x\sqrt{x^2-4}}$$

Use trig substitution, with $x = 2\sec(\theta)$. (Converges to $\pi/4$))

57. Straightforward, but note that, as $t \to 0^+$, the expression t^{1-p} will diverge if $1 - p$ is negative.

71. For part (a), note that $\int e^{-st} \, dt$ is $-e^{-st}/s$. Something similar for part (b) and (c).