7.2 3, 7, 11, 13, 19, 21, 23, 33, 35, 39, 56, 61, 63

#3 $\int_0^{\pi/2} \sin^7\theta \cos^5\theta \, d\theta$: choose either for substitution.

Choosing $u = \sin\theta$, $du = \cos\theta \, d\theta$

we have: $\theta = 0 \Rightarrow \sin(0) = 0$ and $\theta = \pi/2 \Rightarrow \sin(\pi/2) = 1$

$$\int \sin^7\theta \cos^4\theta \, (\cos\theta \, d\theta) = \int \sin^7\theta \, (1-\sin^2\theta)^2 (\cos\theta \, d\theta)$$

$$= \int_0^1 u^7 (1-u^2)^2 \, du = \int_0^1 u^7 (1 - 2u^2 + u^4) \, du$$

$$= \int_0^1 u^7 - 2u^9 + u^{11} \, du$$

$$= \frac{1}{8}u^8 - \frac{1}{5}u^{10} + \frac{1}{12}u^{12} \Big|_0^1$$

$$= \frac{1}{8} - \frac{1}{5} + \frac{1}{12} = \frac{15 - 24 + 10}{120} = \frac{1}{120}$$

7. $\int_0^{\pi/2} \cos^2\theta \, d\theta = \int_0^{\pi/2} \frac{1 + \cos 2\theta}{2} \, d\theta$

$$= \frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) \Big|_0^{\pi/2}$$

$$= \left(\frac{\pi}{4} + \frac{1}{4}\sin(\pi)\right) - (0) = \pi/4$$

11. $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx = \int_0^{\pi/2} \frac{(1 - \cos 2x)}{2} \frac{(1 + \cos 2x)}{2} \, dx$

$$= \frac{1}{4}\int_0^{\pi/2} 1 - \cos^2(2x) \, dx = \frac{1}{4}\int_0^{\pi/2} 1 \, dx - \frac{1}{4}\int_0^{\pi/2} \frac{1 + \cos(4x)}{2} \, dx$$

$$= \frac{\pi}{8} - \frac{1}{8}\left(x + \frac{1}{4}\sin(4x)\right) \Big|_0^{\pi/2}$$

#11, cont.

$$\frac{\pi}{8} - \frac{\pi}{16} = \frac{\pi}{8}$$

#13 $\int t \sin^2(t)\, dt = \int t \left(\frac{1-\cos(2t)}{2} \right) dt$

$= \frac{1}{2}\int t\, dt - \frac{1}{2}\int t \cos(2t)\, dt$

$= \frac{1}{2}t^2 - \frac{1}{2}[\quad]$

+	t	$\cos(2t)$
−	1	$\frac{1}{2}\sin(2t)$
+	0	$-\frac{1}{4}\cos(2t)$

$= \frac{1}{2}t^2 - \frac{1}{2}\left(\frac{1}{2}t \sin(2t) + \frac{1}{4}\cos(2t)\right) + C$

#19 $\int \frac{\cos(x) + \sin(2x)}{\sin x} dx = \int \frac{\cos(x) + 2\sin(x)\cos(x)}{\sin(x)} dx =$

$\int \underbrace{\frac{\cos x}{\sin x}}\, dx + 2\int \cos(x)\, dx = \ln|\sin(x)| + 2\sin(x) + C$

$u = \sin x$
$du = \cos x\, dx$

$\int \frac{1}{u} du = \ln|u|$

#21 $\int \tan(x) \sec^3 x\, dx \Rightarrow$ Reserve $\sec x \tan x\, dx$ for substitution.

$\int \sec^2 x\, (\sec x \tan x\, dx) = \int u^2 du = \frac{1}{3}u^3 + C$

$= \frac{1}{3}\sec^3 x + C$

23. $\int \tan^2 x \, dx = \int \sec^2 x - 1 \, dx = \tan x - x + C$

33. $\int x \sec x \tan x \, dy$

$+$	x	$\sec(x) \tan x$
$-$	1	$\sec(x)$
$+$	0	$\int \sec x \, dx$

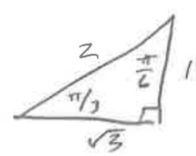
$\hookrightarrow \ln|\sec x + \tan x)$

$\Big\}\; x \sec x - \ln|\sec x + \tan x| + C$

35. $\int_{\pi/6}^{\pi/2} \cot^2 x \, dx$ Compare with $\tan(x)$, and note that

$\sin^2 x + \cos^2 x = 1 \Rightarrow 1 + \cot^2 x = \csc^2 x$

$$\int_{\pi/6}^{\pi/2} \csc^2 x - 1 \, dx = -\cot(x) - x \Big|_{\pi/6}^{\pi/2}$$



$$= -\cot(\pi/2) - \pi/2 + \cot(\pi/6) - \pi/6$$

$$= 0 - \pi/2 + \sqrt{3} + \pi/6$$

$$= \sqrt{3} - \pi/3$$

39. $\int \csc x \, dx$ Same trick as $\sec(x)$ -

$$\int \csc(x) \frac{\csc(x) + \cot(x)}{\csc(x) + \cot(x)} \, dx$$

If $u = \csc(x) + \cot(x)$

$du = -\csc(x) \cot x - \csc^2 x \, dx$

$\Big\}\; -\int \frac{1}{u} \, du = -\ln|\csc(x) + \cot(x)| + C$

Note on #39: The text multiplies top and bottom by $\csc(x) - \cot(x)$ so that their soln is

$$\ln|\csc(x) - \cot(x)| + C$$

As it turns out, this is the same as

$$-\ln|\csc(x) + \cot(x)| + C$$

but that is not obvious.

#54 $\int \sin x \cos x \, dx$ 4 ways:

1) $u = \cos(x)$ $du = -\sin(x)\,dx$, and

$$-\int \cos(x)(-\sin x)\,dx = -\int u\,du = -\tfrac{1}{2}\cos^2 x + C_1$$

2) $u = \sin(x)$ $du = \cos(x)\,dx$, and

$$\int u\,du = \tfrac{1}{2}u^2 + C_2 = \tfrac{1}{2}\sin^2 x + C_2$$

3) $$\int \sin x \cos x\,dx = \tfrac{1}{2}\int \sin(2x)\,dx = -\tfrac{1}{4}\cos(2x) + C_3$$

4) IBP

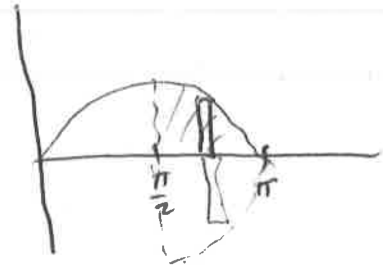
$$\begin{array}{ccc} + & \cos(x) & \sin(x) \\ - & -\sin(x) & -\cos(x) \end{array} \Rightarrow \begin{array}{l} \int \sin(x)\cos(x)\,dx = \\ \cos^2(x) - \int \sin(x)\cos(x)\,dx \end{array}$$

so $$2\int \sin(x)\cos(x)\,dx = \cos^2(x)$$

$$\int \sin(x)\cos(x)\,dx = \tfrac{1}{2}\cos^2(x) + C_4$$

You can see the relationship by substituting $1 - \sin^2(x)$ in for $\cos^2(x)$ in #1, for example.

#61 Using disks, $r = \sin(x)$



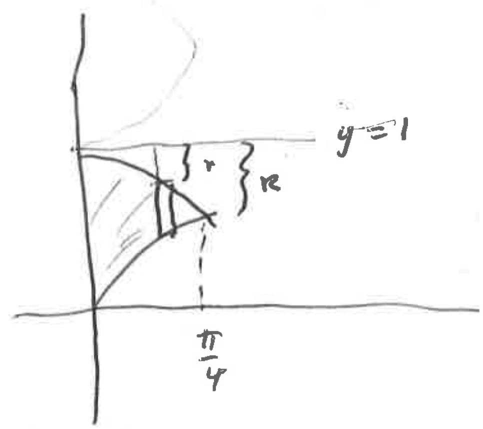
so

$$V = \pi \int_{\pi/2}^{\pi} \sin^2(x)\,dx$$

$$= \frac{\pi}{2}\int_{\pi/2}^{\pi} (1-\cos(2x))\,dx$$

$$= \frac{\pi^2}{4}$$

#63 Use washers:



$R = 1-\sin(x)$
$r = 1-\cos(x)$

so $$V = \pi\int_0^{\pi/4} \left((1-\sin(x))^2 - (1-\cos(x))^2 \right) dx$$

$$= \pi \int_0^{\pi/4} 2\cos(x) - 2\sin(x) + \underbrace{\sin^2 x - \cos^2(x)}_{\text{use } -\cos(2x)\,?}\,dx$$