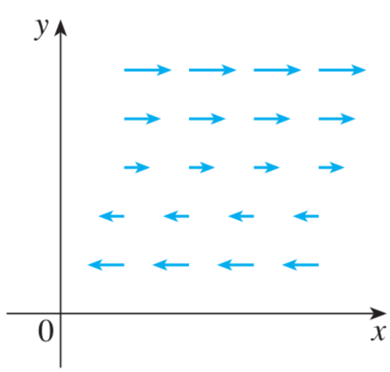
Final Exam Pack B

1. Short Answer/True or False.
   (a) True/False: The linear equation $Ax + By + Cz + D = 0$ represents a line in space.
   (b) True/False: If $|\mathbf{r}(t)|^2 = k$ for all t, then $\mathbf{r}$ and $\mathbf{r}'$ are orthogonal.
   (c) If $\mathbf{r}(t) = \langle \sqrt{2-t}, \ln(t-1) \rangle$, then find the domain of $\mathbf{r}$, and compute $\mathbf{r}'$.
   (d) Find the differential of $u = \sqrt{x^2 + 3y^2}$.
   (e) True/False: If $\int_C \mathbf{F} \cdot d\mathbf{r}$ when C is the unit circle, then $\mathbf{F}$ is conservative.
2. Let vector $\mathbf{a} = \langle 2, 1, 3 \rangle$ and $\mathbf{b} = \langle -1, 2, 0 \rangle$.
   (a) Find the area of the parallelogram formed using $\mathbf{a}, \mathbf{b}$ (as position vectors).
   (b) Find the component of the projection of $\mathbf{a}$ onto $\mathbf{b}$ in the direction of $\mathbf{b}$. That is, $\text{comp}_{\mathbf{b}}(\mathbf{a})$.
3. Find the distance between the plane $2x - 3y + z = 4$ and $4x - 6y + 2z = 3$.
4. Find the limit, if it exists: $\lim_{(x,y)\to(0,0)} \frac{xy^2}{x^2+y^4}$
5. Reparameterize the curve with respect to arc length: $\mathbf{r}(t) = \langle 2t, 1-3t, 5+4t \rangle$.
6. Let $f(x, y) = 4 - x^2 - y^2$. Find $f_y(2, 1)$ using the *definition* of the partial derivative.
7. $u = xe^{ty}$ where $x = \alpha^2\beta, y = \beta^2\gamma, t = \gamma^2\alpha$. Find $\partial u/\partial\beta$ if $\alpha = -1, \beta = 2, \gamma = 1$.
8. Find the maximum rate of change of f at the given point and the direction in which it occurs: $f(x, y, z) = (x + y)/z$, at $(1, 1, -1)$.
9. Let $f(x, y) = xy$. Find $\nabla f(3, 2)$ and use it to find the tangent line to the level curve $f(x, y) = 6$ at the point $(3, 2)$. Sketch the level curve, the tangent line and the gradient vector.
10. Find three positive numbers whos sum is 12 and whose product is a maximum.
11. Sketch the region of integration and change the order: $\int_{-2}^{2} \int_{0}^{\sqrt{4-y^2}} f(x, y) dx\, dy$
12. Given the following triple integral, rewrite using two other orders of integration (your choice):

$$\int_0^2 \int_0^{y^3} \int_0^{y^2} f(x, y, z)\, dz\, dx\, dy$$

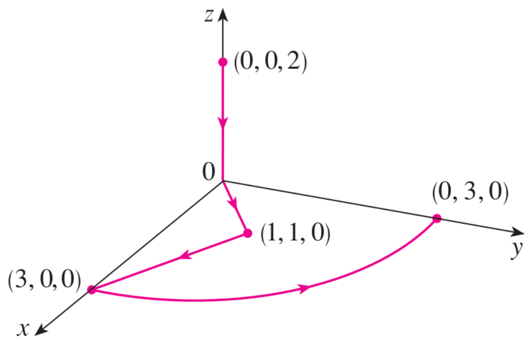
13. Find the work done by the vector field $\mathbf{F} = \langle (1 + xy)e^{xy}, x^2e^{xy} \rangle$ in moving a particle CCW along the ellipse $x^2 + y^2/4 = 1$ (that is, from point $(1, 0)$ to $(0, 2)$).
14. For the vector field $\mathbf{F} = \langle P, Q \rangle$ below, estimate the signs (positive, negative, zero) of each: (i) P_x, P_y, (ii) Q_x, Q_y, then (iii) the curl and divergence.



15. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, if

$$\mathbf{F} = \langle 3x^2yz - 3y, x^3z - 3x, x^3y + 2z \rangle$$

And C is the path shown below, starting at $(0, 0, 2)$ and ending at $(0, 3, 0)$.



16. Set up, but do not evaluate, the flux integral for $\mathbf{F} = \langle -y, 0, 1 \rangle$ across the surface $\mathbf{r}(u, v) = \langle uv, u^2, u - 2v \rangle$ where $0 \leq u \leq 1$ and $0 \leq v \leq 1$.

17. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, if $\mathbf{F} = \langle yz, 2xz, e^{xy} \rangle$, where C is the circle (CCW) $x^2 + y^2 = 16$, $z = 5$.

18. Find the work of the vector field $\mathbf{F} = \langle 2z, 8x - 3y, 3x + y \rangle$, moving a particle along C is the boundary of the part of the plane going through (in order) $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 2)$, then back to $(1, 0, 0)$.

19. Find the flux of $\mathbf{F} = \langle 3x - 2y + 4z \rangle$ across the surface of the sphere $x^2 + y^2 + z^2 = 9$.