Example in-class

$$f(x,y) = 2x^2 + 3y^2 - 4x - 5$$

on $\quad x^2 + y^2 = 16.$

Compute the gradients:

$$\nabla f = \langle 4x-4,\ 6y \rangle$$

$$\nabla g = \langle 2x,\ 2y \rangle$$

So:
$$4x - 4 = 2\lambda x$$
$$6y = 2\lambda y \Rightarrow \text{If } y \neq 0,\ \lambda = 3$$
$$x^2 + y^2 = 16$$

If $\lambda = 3$, from Eqn 1, $4x - 4 = 6x$, so $x = -2$, and $y = \pm 2\sqrt{3}$.

We said in class that $f(-2, \pm 2\sqrt{3}) = 47$.

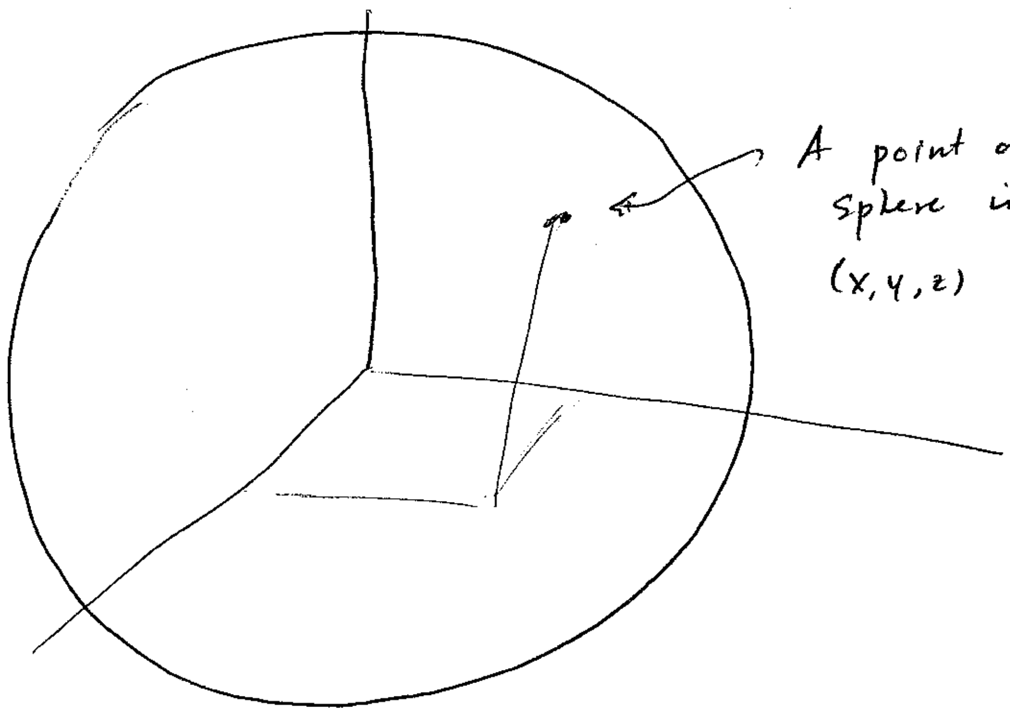
However, we did not finish it all the way.

If $y = 0$, then $x = \pm 4$, and

$$f(-4,0) = 43 \quad \text{and} \quad f(4,0) = 11$$

The global min

Exercise 45, 14.7



The box has length $2x$, width $2y$ and height $2z = 2\sqrt{r^2 - x^2 - y^2}$

$$V(x,y) = 8xy\sqrt{r^2 - x^2 - y^2}, \quad 0 < x < r, \ 0 < y < r$$

$$V_x = \frac{8y\,(r^2 - 2x^2 - y^2)}{\sqrt{r^2 - x^2 - y^2}} \qquad V_y = \frac{8x\,(r^2 - x^2 - 2y^2)}{\sqrt{r^2 - x^2 - y^2}}$$

$$\nabla V = 0 \Rightarrow 2x^2 + y^2 = r^2 \quad \text{and} \quad r^2 = x^2 + 2y^2$$

$$\Rightarrow 2x^2 + y^2 = x^2 + 2y^2 \Rightarrow x^2 = y^2 \Rightarrow \cancel{xy} \quad x = y.$$

$$\therefore \ x = \frac{r}{\sqrt{3}}, \quad y = \frac{r}{\sqrt{3}}, \quad z = \sqrt{r^2 - \frac{r^2}{3} - \frac{r^2}{3}}, \ \text{etc.}$$