GROUP WORK 4, SECTION 12.3

Find the Error (Part 1)

It is a beautiful spring morning. You are about to go to your 10 A.M. class, but have stopped at a convenience store to buy carrot sticks and bottled water for a healthy snack. As you wait in line to pay for your purchases, whistling to yourself, you notice a familiar wild-eyed gentleman standing in line in front of you, buying a moon pie.

"Well aren't you a merry grig?" he asks. You nod noncommittally, since you have no idea what a grig is. He takes your nod to mean that you would like further conversation, and asks, "Where are you off to now?"

"Why, I'm off to my calculus class, to learn some useful information about vectors."

"Vectors, vectors," he says, half to himself. "I remember learning about vectors... I remember learning... LIES!"

"What do you mean, 'lies'?" you ask. "Everything we've learned about vectors is as true as it is useful."

"Oh yes? You think you know it all, do you?" By this point, he has paid for his purchases. As you pay for yours, you notice him writing on his receipt:

Let $\mathbf{u}$ be a vector such that $|\mathbf{u}| = \sqrt{2}$. Choose a vector $\mathbf{v} \neq \mathbf{u}$ such that $\mathbf{u} \cdot \mathbf{v} = 2$. Now we have

$$\begin{aligned}
2(\mathbf{u} \cdot \mathbf{u}) &= \mathbf{u} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v} \\
2(\mathbf{u} \cdot \mathbf{u}) - 2(\mathbf{u} \cdot \mathbf{v}) &= \mathbf{u} \cdot \mathbf{u} + \mathbf{u} \cdot \mathbf{v} - 2(\mathbf{u} \cdot \mathbf{v}) \\
2(\mathbf{u} \cdot \mathbf{u}) - 2(\mathbf{u} \cdot \mathbf{v}) &= \mathbf{u} \cdot \mathbf{u} - \mathbf{u} \cdot \mathbf{v} \\
2\mathbf{u} \cdot (\mathbf{u} - \mathbf{v}) &= \mathbf{u} \cdot (\mathbf{u} - \mathbf{v}) \\
2\mathbf{u} &= \mathbf{u} \\
2 &= 1
\end{aligned}$$

"I've seen this before," you say dismissively, "in college algebra. You are not allowed to divide by zero."

"Ah, but I am not dividing by zero! Since $\mathbf{u} \neq \mathbf{v}$, we know that $\mathbf{u} - \mathbf{v}$ cannot be zero! Now you go enjoy your teacher's lie, while I enjoy my moony pie!" And the stranger leaves, singing a strange song to himself and opening his moon pie wrapper.

Has he finally found a fatal flaw in your class? Two can't equal one, can it? Find the error in the gentleman's reasoning.