

## GROUP WORK 4, SECTION 12.3

### Find the Error (Part 2)

After determining the stranger's mistake, you go to your calculus class. Your teacher tells you to pay particular attention to page 780, so you take a scrap of paper to mark the page. You notice that you are using the gentleman's receipt, and that he has written something on the back as well!

1 moon pie                      \$1.50

Dear Merry Grig,

If I haven't already convinced you that your teacher is nothing but a purveyor of falsity, check this out:

Let  $\mathbf{u}$  be a vector such that  $|\mathbf{u}| = 1$ . Choose a vector  $\mathbf{v}$  such that  $\mathbf{u} \cdot \mathbf{v} = 3$  and  $|\mathbf{v}| = \sqrt{5}$ . Now we have

$$\begin{aligned} |\mathbf{u} - \mathbf{v}|^2 &= (\mathbf{u} - \mathbf{v}) \cdot (\mathbf{u} - \mathbf{v}) \\ &= \mathbf{u} \cdot \mathbf{u} - 2(\mathbf{u} \cdot \mathbf{v}) + \mathbf{v} \cdot \mathbf{v} \\ &= 0 \end{aligned}$$

Hence  $\mathbf{u} = \mathbf{v}$ , since  $\mathbf{u} - \mathbf{v} = \mathbf{0}$ . But  $\mathbf{u}$  and  $\mathbf{v}$  have different lengths!

Well, gosh darn him anyway! How can two things be the same, and yet different?

Find the error.