Limits and Partial Derivatives in Maple

Multivariate Limits

Multivariate limits can be difficult to compute by hand- Maple can have a difficult time as well. However, often Maple is able to compute a multivariate limit- and we use the `limit` command we learned earlier. There are some excellent examples in the help file: `?limit`

Examples

1. Find $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3x^2 y}{x^2 + y^2}$, if it exists.

   You should look at the plot first to see if you can first guess.

   SOLUTION in Maple:

   ```
   F:=3*x^2*y/(x^2+y^2);
   plot3d(F,x=-1..1,y=-1..1)
   ```

   We see a nice surface- It may have some edges, but nothing that should give a discontinuity. See if Maple can find the limit:

   ```
   limit(F,{x=0,y=0});
   ```

   Maple does find the limit is zero. Algebraically, we can back this up using the Squeeze Theorem:
   $$x^2 \leq x^2 + y^2 \quad \Rightarrow \quad \frac{x^2}{x^2 + y^2} \leq 1 \quad \Rightarrow \quad \frac{3x^2 y}{x^2 + y^2} \leq 3|y|$$

   And with a similar argument, the function is bounded below by $-3|y|$, which we can verify with a plot:

   ```
   plot3d({-3*abs(y),F,3*abs(y)},x=-1..1,y=-3..3);
   ```

   Therefore, by the Squeeze Theorem, the limit is 0.

2. Show that the following limit either exists or show that it does not exist:
   $$\lim_{(x,y)\to(0,0)} \frac{y^2 \sin^2(x)}{x^4 + y^4}$$

   SOLUTION: First plot it to see what it looks like:

   ```
   G:=y^2*(sin(x))^2/(x^4+y^4);
   plot3d(G,x=-1..1,y=-1..1,shading=zhue,axes=boxed);
   ```

If you go to the origin along a purple "valley", the limit will be zero. This would be, for example, along either the $x-$ or $y-$axis:

$$\lim_{(x,y)\to(0,0)} \frac{y^2 \sin^2(x)}{x^4 + y^4} = 0$$

However, trying to go to the origin along the orange "hill", $y = x$, looks like a different limit. Let's try it: Along $y = x$, the expression becomes

$$\frac{x^2 \sin^2(x)}{2x^4} = \frac{\sin^2(x)}{2x^2} = \frac{1}{2}\left(\frac{\sin(x)}{x}\right)^2$$

You may recall that the limit in the parentheses is 1 (l'Hospital's rule, for example), so that the overall limit is now 1/2- which agrees with our graph.

Therefore, the limit at the origin does not exist, and this function is not continuous at the origin.

This is typical of this kind of discontinuity.

3. Does this limit exist?
$$\lim_{(x,y)\to(0,0)} \frac{x^4 - 4y^2}{x^2 + 2y^2}$$

SOLUTION: Plot in Maple, and you'll see something similar to the last example- This limit does not exist.

```
plot3d((x^4-4*y^2)/(x^2+2*y^2),x=-1..1,y=-1..1,shading=zhue,axes=boxed);
```

From the graph, we see that if we travel along the $y-$axis, the limit is -2, but if we travel along the $x-$axis, the limit is 0 (we would want to actually show the algebra as well).

Derivatives and Partial Derivatives

Partial derivatives are easy in Maple, as are higher derivatives.

- **EXAMPLE:** Given $f(x, y) = \sqrt{x^2 + y^2}$, find f_x, f_{yx} and f_{yyx}

  SOLUTION: (Note that we're differentiating expressions and not functions)

```
f:=sqrt(x^2+y^2);
fx:=diff(f,x);
fyx:=diff(f,y,x);
fyyx:=diff(f,y,y,x);
```

Is f_{yyx} the same as f_{yxy}? Check in Maple.

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- Suppose $u(x, y) = e^x \sin(y)$. Show that u satisfies "Laplace's Equation":

$$u_{xx}(x, y) + u_{yy}(x, y) = 0$$

SOLUTION: In Maple, this is easy to check:

```
u:=exp(x)*sin(y);
uxx:=diff(u,x,x);
uyy:=diff(u,y,y);
uxx-uyy;
```