Week 6, Calc Lab

Solve the following in a Maple Worksheet. Upload the worksheet to CLEO before our next class meeting. You may work in groups of up to three, but each person should have their name on the worksheet, and each student should upload their own copy of the worksheet.

The answers to most of the questions below will be found in our Maple documents.

1. Plot the $(4,3)-$torus knot in three dimensions, given parametrically by:
$$x(t) = \cos(3t)(3 + \cos(4t)), \quad y(t) = \sin(3t)(3 + \cos(4t)), \qquad z(t) = \sin(4t)$$

   The knot is a closed curve, meaning that the curve comes together and starts to repeat itself. Find the arc length of the knot.

   Hint 1: There is a built-in `ArcLength` command. Use Maple's "help" facilities to figure out how to use it.

   Hint 2: You'll need a starting and ending time. You might start by guessing from the graph. Double check by evaluating x, y, z at these times (using Maple- Use the `subs` command).

2. Consider the curve: $y = 1 + \sin(x)$, $0 \le x \le 3\pi$. Construct the parametric surface representing the curve rotated about the $x-$axis, then have Maple plot it.

3. Consider the surface:
$$\langle u\cos(v), u\sin(v), u \rangle \qquad -1 \le u \le 1, \quad 0 \le v \le 2\pi$$

   (a) Plot the surface in 3d.
   (b) If v is held constant, what kinds of curves do we get in 3d? (Answer as a comment)
   (c) If u is held constant, what kinds of curves do we get in 3d? (Answer as a comment)
   (d) Could this surface be a surface of revolution? What function is being rotated, and around what axis?

4. Find the limit, if it exists, or show that it does not exist:

   (a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 - 4y^2}{x^2 + 2y^2}$

   (b) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x^2 + 3xy + 4y^2}{3x^2 + 5y^2}$

5. Is the function f continuous? Why or why not?
$$f(x,y) = \frac{1 + x^2 + y^2}{1 - x^2 - y^2}$$

6. Is the function f continuous? Why or why not?
$$f(x,y) = \begin{cases} \frac{\sin(xy)}{xy} & xy \ne 0 \\ 1 & xy = 0 \end{cases}$$